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Keywords: income distribution, consumer debt, emulation, instability, limit cycle
Income distribution, consumer debt, and keeping up with the Joneses: a Kaldor-Minsky-Veblen model

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Abstract

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1. Introduction

The US consumption expansion, accompanied with significant household debt accumulation, from the 1980s to the early 2000s, was a substantial

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source of macroeconomic stimulus. However, the rapid accumulation of household debt turned out to be unsustainable, as evidenced by the Great Recession of 2007–2009.

The importance of household debt in the recent financial crisis seems indisputable but the question remains to be addressed what caused household indebtedness to reach the unprecedented level. Among other explanations, there has been a growing literature that stresses rising income inequality as a root-cause of the rising indebtedness (Cynamon and Fazzari, 2008; Barba and Pivetti, 2009; Foster and Magdoff, 2009; Rajan, 2010; Setterfield, 2013). According to the literature, lower-income households who experienced the secular decline in their income share had to rely more on debt financing in order to maintain their desired consumption. While a majority of contemporary economists guided by rational expectations hypothesis may see this development as a result of consumption smoothing by forward-looking agents the burgeoning literature has provided alternative perspectives.

Cynamon and Fazzari (2008), for instance, argue that household borrowing “did not necessarily correspond to a careful plan for repayment consistent with forward-looking intertemporal budget constraints....an explanation of the recent consumption and debt trends requires us to move beyond atomistic, representative consumers to consider fundamental social influences on household spending and financial decisions.” In particular, a social reference group is a key determinant for consumption behavior. Moreover, advertising and sales promotion in the mass media tend to expand the reference group for lower-income households beyond their peers to include the upper income class. The lower income households’ desire to emulate the richer may provide a strong motivation for borrowing to finance their desired consumption. Thus Barba and Pivetti (2009) maintain that the desire to keep up with the wealthier, coupled with increasing income inequality, was a main cause of household debt accumulation in the US. The notion of emulation in consumption behavior has a long history, at least dating back to Veblen (1912) and Duesenberry (1967), and its importance was also stressed by some

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2 Krueger and Perri (2006) represents a classic example of this perspective.
3 Ravina (2005) finds that, based on the estimation of the Euler equations for a representative sample of U.S. credit card account holders, the consumption of the reference group is an important determinant of household consumption behavior. In her study, the level of per capita consumption of the city where the household lives was used to measure the consumption level of the reference group.

Household debt might not have grown to the unprecedented level if profound changes in the financial sector had not occurred. The introduction of new financial products and practices has reduced lending standards and provided easier access to consumer loans. Securitization, for instance, has contributed to the deterioration of general lending standards and credit expansion (Shin, 2009; Keys et al., 2010; Kara et al., 2011). Financial deregulation also played an important role in the expansion of household debt. According to Burhouse (2003), the deregulation process of consumer lending started in the late 1970s and early 1980. The state usury restrictions were dismantled during this time, and it led the way to remove the interest rate restrictions. Credit card lenders were among the first to respond to the removal of interest rate restrictions and expanded the general purpose credits card across the country.

We believe that all these factors—increasing inequality, consumption emulation and easier access to credit—are crucial in understanding rising household debt in the run-up to the financial crisis. The purpose of this paper is to examine the mechanism of instability and cycles driven by the interaction among those factors. In so doing, we present a macrodynamic model with two distinct features. First, our model focuses on the implications of changes in functional income distribution for consumption and debt dynamics. In our model, functional income distribution is determined endogenously by the level of aggregate demand along the lines of Kaldor’s Keynesian theory of income distribution (Kaldor, 1956). Second, our model

\footnote{Emulation behavior may affect the labor supply decision. Bowles and Park (2005) find a positive relationship between work hours and income inequality using the data of ten advanced economies and interprets that this relationship is due to the desire of those less well off to emulate the consumption standard of rich. Our study does not address this channel and focuses on consumption emulation.}

\footnote{For example, Bayoumi (1993) provides evidence that financial deregulation caused a fall in saving rate in UK during the 1980s. The ratio of total outstanding consumer credit to GDP and a dummy variable representing the sudden financial deregulation (so called the Big Bang) in 1986 in London were used as proxies for financial deregulation.}

\footnote{The key event was a supreme court decision in Marquette National Bank of Minneapolis v.s. First Omaha Service Corp. This ruling allowed the interest rate restriction from the lenders’ state to be applied regardless of the borrowers’ residential states’ law. This effectively led the way for states to remove their interest rate restrictions to be attractive to the lenders (Ellis, 1998; Burhouse, 2003).}
explicitly introduces the low-income households’ desire to emulate the richer. We assume that the emulation motive influences the demand for credit which is partially accommodated by the banking system.

These two features distinguish our study from other studies. For example, Krueger and Perri (2006) and Iacoviello (2008) focus on within-group inequality defined as a rise in the variance of individual incomes. According to these studies, household optimizing behavior to smooth consumption in the face of increasing within-group income inequality explains some important stylized facts such as (i) moderate changes in consumption inequality (relative to the notable increase in income inequality) and (ii) rising household indebtedness since the early 1980s. Unlike the focus on within-group inequality in these studies, Kumhof and Rancière (2010) concern the implications of between-group inequality, more specifically, an increase in the labor income share of the rich class caused by the exogenous rise in their bargaining power. Given the increase in the bargaining power of the rich, optimal consumption decisions of the two classes result in increases in the ratio of debt to income of the poor. Kumhof and Rancière assume that the increase in the debt to income ratio of the poor raises ‘the probability of crisis’ which destroys a fraction of aggregate capital stock. Our model has a similarity to Kumhof and Rancière (2010) in its emphasis on between-group inequality but in our model the state of income distribution is determined endogenously through its interaction with aggregate demand. In addition, the emulation motive, a crucial element of our model, is absent in Kumhof and Rancière (2010) as well as in Krueger and Perri (2006) and Iacoviello (2008).

Two key findings of our analysis are to be highlighted. First, our analysis shows that the combination of the two central features of the model – the Kaldorian mechanism of income distribution and the Veblenian pecuniary

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7 Krueger and Perri (2006), using the data from the Consumer Expenditure Survey, find that the standard deviation of log consumption rose only by half as much as that of income in the US between 1980 and 2003. The similar finding is also provided by Heathcote et al. (2010).

8 The model assumes the existence of two classes in which the one class does not own any capital stock and intertemporal optimization makes them ultimate borrowers given the structure of the model. The division of labor income between the two classes is modeled by a simple Nash bargaining process.

9 Due to this special assumption, high indebtedness can have negative welfare implications in Kumhof and Rancière (2010) unlike Krueger and Perri (2006) and Iacoviello (2008).
emulation – provides a strong source of instability and cycles. Second, the stability property of the system depends crucially on the strength of the emulation motive and the nature of the financial system. We find that instability is more likely when the workers’ desire to emulate the rich is stronger and when banks are more accommodating and place less emphasis on the borrowers’ income status in their lending decisions.

The paper is organized as follows. Section 2 presents the general assumptions of our model and section 3 discusses Kaldor’s Keynesian theory of income distribution in Kaldor (1956). Section 4 sets out basic elements of our model and characterizes temporary equilibrium. Sections 5 and 6 examine the conditions under which instability and cycles arise from the interaction between debt and emulation dynamics. Section 7 offers concluding comments.

2. General assumptions

A central feature of our model is that fluctuations in aggregate demand determine profit margins and therefore drive endogenous variations in income distribution. A consumption boom, for instance, raises aggregate demand which is absorbed by an increase in the markup and the profit share. This idea dates back to Keynes (1930) and Robertson (1933) but receives a more formal treatment in Kaldor (1956). There are assumptions underlying this approach in regard to firms’ pricing/output decisions, accumulation behavior, technology and the labor market.

Concerning firms’ pricing/output decisions, a majority of post-Keynesian models assume that output instantaneously adjusts to clear the goods market for a given markup but our model assumes the opposite: the adjustment of output is costly and therefore sluggish; at any moment of time excess demand or supply in the goods market is eliminated via the adjustment of the markup (thus the profit share) for a given level of output. Therefore income distribution is determined endogenously so as to bring savings in line with investment\(^{10}\).

The economy is mature in the sense that the availability of labor supply constrains output expansion (Kaldor, 1966). We assume that the labor force grows at a constant rate, \(n\).

\(^{10}\)The same mechanism is used in a short-run context by Hahn (1951).
Regarding production technology, homogeneous output is produced by capital and labor with fixed coefficients which implies that, in the absence of labor hoarding, the level of output is proportional to the level of employment. Productive capacity may not be fully utilized at all times and excess capacity may be desired due to strategic reasons including entry deterrence (Spence, 1977). Since the technical output-capital ratio is fixed by assumption, the actual output-capital ratio \( u(t) \) can be used as a measure of capacity utilization\(^{11}\):

\[
\frac{u(t)}{K(t)} = \frac{Y(t)}{K(t)} \tag{1}
\]

where \( Y(t) \) is aggregate output and \( K(t) \) aggregate capital stock.

Following the Keynesian tradition, our model assumes that investment and saving decisions are made independently of each other and therefore investment is not passively determined by savings. More specifically, we adopt a Harrodian accumulation assumption (Harrod, 1939). According to the Harrodian perspective, firms aim to maintain a desired rate of utilization and the discrepancy between the actual and the desired utilization rates motivates firms to speed up or to slow down capital accumulation\(^{12}\). Because firms’ demand expectations are not always met and capital stocks adjust slowly in the short run, the actual rate of utilization may deviate from the desired rate. In the longer run, however, the actual rate cannot persistently deviate from the structurally desired rate since capital stocks can adjust to maintain the desired rate. Abstracting from short-run fluctuations of actual utilization rates, the long-run average rate of utilization, denoted as \( \pi(t) \), can be approximated by the desired rate \( u^* \).

\[
\pi(t) = u^* \tag{2}
\]

where the desired rate \( u^* \) is taken as constant for simplification. Because we are interested in studying the dynamic interaction between consumption and debt accumulation over a longer period of time, we leave out the issues of short-run business cycles. The relevant time horizon in our study is much

\(^{11}\)Denoting the technical output-capital ratio as \( \sigma \), full-capacity output can be written as \( Y^F(t) = \sigma K(t) \). The degree of capacity utilization is proportional to the actual output-capital ratio: \( u(t)/Y^F(t) = Y(t)/(K(t)\sigma) = u(t)/\sigma \).

\(^{12}\)The Harrodian accumulation assumption plays a central role in the Skott’s Kaldorian model of business cycles (Skott, 1989).
longer than that in conventional business cycle analyses. Therefore it is sufficient to refer to the long-run average rate of utilization rather than the actual rate in our analysis.

Denoting as $\bar{g}(t)$ the long-run average rate of accumulation, investment can be written as

$$I(t) = (\bar{g}(t) + \delta)K(t)$$

where $I(t)$ (trend) real investment, $K(t)$ capital stock and $\delta$ the depreciation rate. The growth rate of capital stock exhibits substantial variations over the short run but in a labor constrained economy the long-run average growth rate of capital stock can be approximated by the natural rate of growth. Thus we introduce another long-run approximation.

$$\bar{g}(t) = n$$

3. Kaldor’s theory of income distribution (Kaldor, 1956)

Kaldor’s model is based on the assumption that the propensity to save out of profits ($s^r$) is greater than that of wages ($s^w$), i.e. $s^w < s^r \leq 1$. The condition for goods market equilibrium requires saving to equal investment:

$$s^w[p(t)Y(t) - \Pi(t)] + s^r\Pi(t) = p(t)I(t) \quad (K1)$$

where $p(t)$ is output price, $Y(t)$ aggregate output and $\Pi(t)$ nominal gross profits. The left-hand side of the equation refers to total saving, the sum of the savings from wages and profits.

Using (1)-(4), (K1) can be solved for the equilibrium profit share:

$$\left( \frac{\Pi(t)}{p(t)Y(t)} \right)^* = \left( \frac{1}{s^r - s^w} \right) \left( \frac{n + \delta}{u^*} - s^w \right) \quad (K2)$$

The following assumption ensures that the equilibrium profit share is greater

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13 The focus on the longer-run is justified in light of styled facts, e.g. a secular rise in household indebtedness for several decades followed by a severe debt crisis.

14 Neither the equality of $\pi(t)$ and $u^*$ or the exogeneity of $u^*$ is necessary to our analysis. Ryoo (2010) discusses the related issues in detail.
than zero and less than one.\footnote{The meaning of (K3) is straightforward. In order for both wage and profit income to be positive, the ratio of gross investment to output $\frac{n + \delta}{u^*}$ must be greater than the propensity to save out of wages and less than that out of profits so as to ensure that the average saving rate takes a value between $s^w$ and $s^r$ in equilibrium.}

\[ s^w < \frac{n + \delta}{u^*} < s^r \]  \hspace{1cm} (K3)

(K2) shows that the equality of saving and investment is achieved through the variation of income distribution. Such an adjustment process implies that the level of output prices in relation to the level of wages is determined by aggregate demand. An autonomous increase in demand – as an increase in $n$, a fall in $s^w$ or $s^r$ – raises output prices and profit margins, thereby shifting income distribution in favor of profit earners.

The distributional effect of aggregate demand in the Kaldor model may generate instability if it is combined with the Veblenian idea of consumption emulation. Suppose that the primary income source of lower-income groups is wages and their individual saving rates are closely correlated with $s^w$. Then assume that individuals in these groups have the desire to keep up with the richer (whose income are largely based on capital income). Such emulation behavior most likely results in a fall in saving propensity out of wages ($s^w$). The decline in $s^w$ represents an increase in aggregate demand, which causes the profit share to increase in the Kaldor model. The increase in the profit share reduces the income share of low-income households. To the extent that worsening income inequality translates into consumption inequality, the initial attempt by low-income households to keep up with the rich may result in a further increase in consumption inequality, which tends to reinforce their desire to increase consumption (and therefore to reduce $s^w$ further). The net result is the continuous decline in both the saving rate and the income share of low-income households. Thus the combination of Kaldor’s theory of income distribution and the Veblenian idea of emulation may produce exploding trajectories. The 1956 Kaldor model, however, does not allow us to pursue this line of reasoning further. The model does not pay attention to financial stocks and their feedback on consumption and distribution. As households make saving or dissaving, they will accumulate assets or debts. Changes in their asset/liability positions as well as associated changes in their incomes likely have implications for their consumption and saving decisions.
in successive periods.

4. Model

We extend the Kaldor model in several directions. Financial stocks are introduced explicitly. Firms issue (or buy back) stocks and retain or distribute their profits to stockholders. Rentiers hold stocks and deposits, with no labor income. The extended model allows workers to accumulate debts. Workers are assumed to be net debtors\(^{16}\), holding no asset.

4.1. Consumption and Borrowing

Workers and rentiers are different in consumption behavior. Workers are credit-constrained and their consumption is affected by bankers’ willingness to extend loans. Rentiers follow a standard consumption behavior: their consumption depends on their income and wealth. The level of consumption of workers and rentiers is given by

\[
p(t)C^w(t) = [p(t)Y(t) - \Pi(t)] - i(t)M(t) + \dot{M}(t) \tag{5}
\]

\[
p(t)C^r(t) = f(Y^r(t), NW^r(t)), \quad 1 > f_1 > 0, \quad f_2 > 0 \tag{6}
\]

where \(C^w(t)\) and \(C^r(t)\) are workers’ and capitalists’ real consumption, \(i(t)\) the nominal interest rate, \(M(t)\) workers’ debt and \(\dot{M}(t)\) net borrowing\(^{17}\). To simplify the analysis we assume that the rate of interest on loans equals that on deposits. \(E\) is the budget constraint facing worker households. The rentiers’ consumption function \(f(\cdot, \cdot)\) is homogeneous of degree one in both arguments. \(f_1\) and \(f_2\) are rentiers’ marginal propensities to consume out of income and wealth\(^{18}\). \(Y^r(t)\) is rentiers’ income which is the sum of dividend and interest incomes. \(NW^r(t)\) is rentiers’ wealth, the sum of stocks and

\(^{16}\)This requires some restrictions over the parameters of the model so that workers’ debts are normally positive. For the time being, we proceed our analysis as if the model satisfy such restrictions over the parameter values.

\(^{17}\)A dot over a variable is used to refer to a derivative of the variable with respect to time throughout the paper.

\(^{18}\)Throughout this paper we use the notational convention so that \(x_2\), for instance, is the first derivative of function \(x\) with respect to its second argument. If the subscript is not a numeral but a letter, then \(\pi_m\), for example, refers to the first derivative of function \(\pi\) with respect to \(m\).
deposits. Formally,

\[ Y^r(t) = (1 - s^f)[\Pi(t) - \delta p(t)K(t)] + rM(t) \]  
\[ NW^r(t) = V(t) + M(t) \]

where \( 1 - s^f \) is the (constant) dividend payout rate, \( r \) the real interest rate and \( V(t) \) the market value of stock holdings. We take the real interest rate \( r \) as exogenous in our model.\(^{20}\) Dividing (5) and (6) by \( p(t)K(t) \), we have:

\[ c^w(t) \equiv \frac{C^w(t)}{K(t)} = y^w(t) + \dot{m}(t) + nm(t) \]  
\[ c^r(t) \equiv \frac{C^r(t)}{K(t)} = f(y^r(t), \omega^r(t)) \]

\[ y^w(t) = [1 - \pi(t)]u^* - rm(t) \]  
\[ y^r(t) = (1 - s^f)[\pi(t)u^* - \delta] + rm(t) \]  
\[ \omega^r(t) = (1 + \alpha)m(t), \quad \text{where } \alpha = V(t)/M(t) \]  
\[ m(t) = \frac{M(t)}{p(t)K(t)} \]

\( \alpha \) is the ratio of rentiers’ stocks to deposit holdings, representing their portfolio preferences but we take it as fixed in this model.\(^{21}\) \( m(t) \) is a measure of workers’ indebtedness, the level of workers’ outstanding debts relative to the size of capital stocks in the firm sector. (9)-(13) show that the consumption of workers and rentiers depends on \( m(t) \) and \( \dot{m}(t) \) and therefore the evolution of workers’ indebtedness is a key determinant of aggregate demand.

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\(^{19}\)We assume that no one holds cash and no bank holds reserves; banking does not incur any cost except their payment obligations to depositors; there is neither corporate nor government debts. Given these assumptions, the amount of rentiers’ deposits equals the amount of bank loans to workers. Therefore we will use the same variable \( M(t) \) to refer to loans and deposits.

\(^{20}\)The nominal rate is related to the real rate by the inflation rate: \( i(t) = r + [p(t)/p(t)] \). We assume that the ex post real rate equals the ex ante rate.

\(^{21}\)Endogenous changes in the portfolio composition can be an important source of instability and cycles is a key element of some formalization of Minsky’s financial instability hypothesis, e.g., Taylor and O’Connell (1985) and Ryoo (2010).
4.2. The banking sector

Banks play a pivotal role in this economy because their lending practices regulate the dynamics of the debt ratio $m(t)$. We introduce the following equation to represent bankers’ lending practices.

$$\dot{m}(t) = \phi(y^w(t), z(t)), \quad \phi_1 > 0, \quad \phi_2 > 0 \quad (15)$$

Equation (15) suggests that the change in the debt ratio depends positively on workers’ net income $y^w(t)$ (wages minus interest paid). Facing default risks in imperfect capital markets, bankers may see borrowers’ current net income $y^w(t)$ as an indicator of their ability to fulfill payment obligations. Using Minsky’s terminology, $y^w(t)$ serves as a margin of safety based on which lending and borrowing activities take place. Minsky argues

...bankers are not simpletons who accept all that is put forward for them to finance as being worthy of financing. In their relations with businessmen, households and governments that require financing, bankers are designated sceptics. [Minsky, 1996][p.76]

Therefore, Minsky argues, banks insist on margins of safety the most important among which is the excess of a unit’s expected operating income over the payment committed by debt contracts. The higher $\phi_1$ the more quickly banks adjust their loans in response to changes in borrowers’ income. In contrast, if $\phi_1 = 0$, bankers do not all take into account borrowers’ income positions.

Credit supply may depend on other factors. While our formulation emphasizes the decisive role of bankers’ ability to constrain credit supply, it seems to be unreasonable to leave out demand-side factors. As a matter of fact, most arguments that relate increased consumption norms to higher indebtedness presuppose the existence of a certain degree of banks’ accommodating behavior, e.g., Cynamon and Fazzari (2008); Barba and Pivetti (2003). In particular, we focus on the workers’ desire to emulate rentiers

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22 The positive dependence of borrowing on net income is also introduced in Dutt (2006).

23 Some literature on the issues of household debt often concerns whether demand or supply factors in credit markets are a main driving force behind the movement of household debt (Pollin [1988], Dutt [2006], Charpe et al. [2012]). Our formulation of debt dynamics is eclectic.

24 Rajan (2010) argues that easy credit was a result of the political reaction to rising inequality since it was perceived as less costly than redistributive tax policies – at least in
as a determinant of credit demand. Thus we introduce a new variable $z(t)$ in order to capture the strength of the workers’ emulation motive. Our specification (15) assumes that changes in the workers’ emulation motive directly translate into the workers’ credit demand which is partially accommodated by bankers. The size of $\phi_2$ represents how much accommodating bankers are when they face the demand for credit by workers. In this regard, $\phi_2$ is related to the level of lending standards. If $\phi_2 = 0$, then bankers never accommodate workers’ demand for credit motivated by their desire to keep up with the rich and therefore the link between debt and emulation dynamics will be severed. Institutional changes and financial innovation that tend to lower banks’ lending standards make the size of $\phi_2$ larger and therefore the banks’ credit supply will respond strongly to changes in the workers’ credit demand. It turns out that the magnitude of both $\phi_2$ and $\phi_1$ has important implications for the stability of the system (see section 6).

4.3. Determination of income distribution

The condition for equilibrium in the goods market is given by

$$Y(t) = C^w(t) + C^r(t) + I(t)$$

Using (1)-(4) and (9)-(13), (16) can be written as:

$$(1 - \pi(t))u^* - rm(t) + \phi[(1 - \pi(t))u^* - rm(t), z(t)] + nm(t) + f[(1 - s^f)(\pi(t)u^* - \delta) + rm(t), (1 + \alpha)m(t)] + n + \delta = u^*$$ (17)

Since $(1 + \phi_1) - f_1(1 - s^f) \neq 0$, the implicit function theorem allows us to write $\pi(t)$ as a function of $m(t)$ and $z(t)$:

$$\pi(t) = \pi(m(t), z(t))$$ (18)

Let us define

$$\Delta \equiv (1 + \phi_1) - f_1(1 - s^f) > 0$$ (19)

(19) is a stability condition that is required to establish the equilibrium profit share in the goods market.25

the short run – in dealing with rising inequality. Such a factor may have strengthened the tendency toward more accommodating lending practices.

25(19) plays the same role as the condition $s^r > s^w$ in the Kaldor model in the previous section.
Let us examine how income distribution and consumption respond to changes in workers’ emulation motives and indebtedness. First, an increase in workers’ credit demand associated with increased emulation motives, \( z(t) \), is partially accommodated by bankers and leads to an increase in actual borrowing. The increase in borrowing fuels consumption demand by workers which raises profit margins.

\[
\frac{\partial \pi(t)}{\partial z(t)} \equiv \pi_z = \frac{\phi_2}{\Delta u^*} > 0 \tag{20}
\]

The positive effect of an increase in \( z(t) \) on the profit share means that the workers’ net income is decreasing in \( z(t) \):

\[
\frac{\partial y^w(t)}{\partial z(t)} \equiv y^w_z = -\frac{1}{\Delta} \phi_2 < 0 \tag{21}
\]

Therefore income distribution adjusts in favor of rentiers as workers take on more loans to keep up with their consumption standards. This link between income distribution and the workers’ desire to debt-finance their consumption implies that the workers’ consumption is decreasing in \( z(t) \).

\[
\frac{\partial c^w(t)}{\partial z(t)} \equiv c^w_z = (y^w_z + \phi_1 y^w_z) + \phi_2 \quad \text{or} \quad -\frac{\phi_2 f_1 (1 - s f)}{\Delta} < 0 \tag{22}
\]

In other words, workers’ attempt to raise their consumption by borrowing, an increase in \( z(t) \), reduces their consumption, \( c^w(t) \). This paradoxical result requires some explanation. The direct effect on the workers’ consumption of a higher credit demand by workers is positive, which is captured by \( \phi_2 \), the last term in (22): bankers partially accommodate the workers’ credit demand and their consumption tends to rise. However there is an induced shift in income distribution against workers, a fall in \( y^w(t) \). The fall in the workers’ net income \( y^w(t) \) reduces their consumption by \( y^w_z \), the first term in (22). Moreover, facing the decline in the borrower’s income, bankers become more reluctant to extend further loans, which is represented by \( \phi_1 y^w_z \), the second term in (22). The negative induced effects of the rise in \( z(t) \) on \( c^w(t) \), \( y^w_z + \phi_1 y^w_z \), dominate the positive direct effect, \( \phi_2 \), making the overall effect
negative.\footnote{26}

The workers’ desire to debt-finance their consumption reduces not only the level but also the share of their consumption. The equilibrium condition for the goods market requires total consumption normalized by capital stocks to be constant:

\[ c^w(t) + c^r(t) = u^* - n - \delta \equiv \bar{c} \]  

(24)

Therefore the negative effect of \( z(t) \) on \( c^w(t) \) means that the workers’ share of consumption, \( c^w(t)/\bar{c} \), falls as \( z(t) \) rises. This has an important implication for the stability of the system (see section 5.2).

Next let us turn to the effect of changes in the debt ratio on distribution and consumption. An increase in workers’ indebtedness \( m(t) \) has an ambiguous effect on the profit share:

\[ \frac{\partial \pi(t)}{\partial m(t)} \equiv \pi_m = \frac{f_1 r + f_2 (1 + \alpha) + n - r (1 + \phi_1)}{\Delta u^*} \geq 0 \]  

(25)

where \( \Delta \equiv (1 + \phi_1) - f_1 (1 - s^f) > 0 \). For a given \( \pi(t) \), the effect of a rise in the debt ratio on lenders’ consumption is unambiguously positive due to an increase in their interest income and wealth \( (f_1 r + f_2 (1 + \alpha) > 0) \) but its effect on workers’ consumption can be either way \( (n - r (1 + \phi_1) \gtrless 0) \), making the overall effect on aggregate demand and the profit share ambiguous.\footnote{27} Thus the required adjustment of the profit share is ambiguous.

Despite this ambiguity, the effect of the debt ratio on the workers’ net income is unambiguously negative.

\[ \frac{\partial y^w(t)}{\partial m(t)} \equiv y_m^w = -\frac{s^f f_1 r + f_2 (1 + \alpha) + n}{\Delta} < 0 \]  

(26)

\footnote{26}Such a dominance of the induced effects over the direct effect is explained by the multiplier effect of demand on distribution. The multiplier mechanism is most transparent if \( \phi_1 = 0 \). An increase in \( z(t) \) raises demand directly by \( \phi_2 \) but reduces the workers’ net income by \( (1/\Delta) \times \phi_2 \) (see (21)). \( 1/\Delta \) is a multiplier in this context, which is unambiguously greater than one.

\footnote{27}The ambiguity of the effect of the debt ratio on aggregate demand could also arise in Kaleckian models (e.g. \textcite{Palley_2010}). In other Kaleckian models (incl. \textcite{Dutt_2006}, \textcite{Dutt_2008} and \textcite{Isaac_and_Kim_Forthcoming}), the short-run effect of higher indebtedness of workers on demand and utilization is unambiguously negative due to the different specification of consumption behavior.
i.e., an increase in workers’ indebtedness reduces workers’ income $y^w(t)$. The intuition is simple. Workers’ income depends not only on their wages but also their debt service payment and an increase in workers’ interest payments due to the rise in $m(t)$ is strong enough to offset the ambiguous effect of $m(t)$ on wages. Finally, the workers’ consumption $c^w(t)$ is decreasing in $m(t)$:

$$\frac{\partial c^w(t)}{\partial m(t)} \equiv c^w_m = (1 + \phi_1)y^w_m + n = -\frac{(1 + \phi_1)[s^f f_1 r + f_2 (1 + \alpha)] + nf_1 (1 - s^f)}{\Delta} < 0 \quad (27)$$

An increase in the debt ratio reduces the workers’ consumption because it decreases their net income $y^w_m < 0$.

### 5. Debt and emulation dynamics

**5.1. Debt dynamics with a constant emulation motive**

Let us consider the dynamics of the debt-capital ratio for a given emulation motive. With $z(t)$ taken as constant, we have a one-dimensional differential equation of $m(t)$.

$$\dot{m}(t) = \phi(y^w(m(t), z(t)), z(t)) \equiv \Phi(m(t), z(t)) \quad (28)$$

To ensure the existence of economically meaningful steady states, we make the following assumption:

**Assumption 1.** For a given value of $z(t)$ in a relevant range,

$$\lim_{m(t) \to 0} \Phi(m(t), z(t)) > 0 > \lim_{m(t) \to \infty} \Phi(m(t), z(t)) \quad (29)$$

Assumption 1 tells us that a debt ratio close to zero is associated with a sufficiently high level of workers’ wage income net of interest paid, causing the debt ratio to increase, whereas a very high debt ratio is associated with a low level of workers’ net income, causing the debt ratio to fall. The first inequality is required for a steady state where workers are in a net debtor position against rentiers.

**Proposition 1.** (Stable debt dynamics under a constant emulation motive)

For a constant emulation motive $z(t)$, the debt dynamics, eq.\[28\], implies that $m(t)$ converges to a unique stationary point $\tilde{m}(z(t))$ such that

$$\phi(y^w(\tilde{m}(z(t)), z(t)), z(t)) = 0 \quad (30)$$
The stationary debt ratio is increasing in the strength of the workers’ emulation motive, i.e., \( \tilde{m}'(z(t)) > 0 \).

**Proof.** Given Assumption 1, the intermediate value theorem ensures the existence of \( m(t) \) in \((0, \infty)\) that satisfies \( \Phi(m(t), z(t)) = 0 \). Uniqueness and stability follows directly from the negative dependence of workers’ income on their indebtedness \( y_m^w < 0 \), which makes the right-hand side of eq[15] decreasing in \( m(t) \):

\[
\frac{\partial \tilde{m}(t)}{\partial m(t)} \equiv \Phi_m = \phi_1 y_m^w < 0 \tag{31}
\]

\( \Phi(m(t), z(t)) = 0 \) implicitly defines the stationary solution as a function of \( z(t) \) and we denote it as \( \tilde{m}(z(t)) \).

\[
\tilde{m}'(z(t)) = -\frac{\Phi_z}{\Phi_m} > 0 \tag{32}
\]

because

\[
\Phi_z = \phi_1 y_m^w + \phi_2 = \frac{\phi_2 [1 - f_1(1 - s_f)]}{\Delta} > 0 \tag{33}
\]

and \( \Phi_m = \phi_1 y_m^w < 0 \). □

The logic behind the stable debt dynamics is clear-cut: an increase in indebtedness reduces the workers’ net income, which induces banks to slow down credit supply.

\( \tilde{m}(z(t)) \) can be interpreted as the workers’ indebtedness that banks are content with. \( \tilde{m}'(z(t)) > 0 \) means that the debt ratio desired by banks is increasing in the workers’ credit demand to debt-finance their consumption. The positive effect rests on the fact that an increase in \( z(t) \), ceteris paribus, accelerates credit supply, i.e. \( \frac{\partial \tilde{m}(t)}{\partial z(t)} = \Phi_z > 0 \) in (33). This requires some explanation. An increase in the workers’ credit demand to finance their consumption, accommodated by bankers, leads to more credit supply. At the same time it reduces the workers’ net income, which makes bankers more reluctant to supply credit. The first effect, \( \phi_2 \), is stronger than the latter, \( \phi_1 y_m^w \), and therefore results in more credit supply, \( \Phi_z > 0 \).

5.2. The evolution of the emulation motive

The stability of debt dynamics in Proposition 1 is contingent upon constant emulation motives \( z(t) \). The worker’s desire to keep up with their consumption norms, captured by \( z(t) \), changes over time. As long as bankers,
at least partially, accommodate workers’ credit demand, i.e., \( \phi_2 > 0 \), changes in \( z(t) \) affect the debt dynamics. Our main objective is to examine the conditions under which the interaction between debt and emulation dynamics leads to instability and cycles.

In our framework, the emulation motive can be formalized by making changes in \( z(t) \) depend on the consumption of an individual worker relative to the average consumption of rentiers. More specifically, let us consider

\[
\dot{z}_j(t) = \tilde{\psi} \left( \frac{c_{w,j}(t)}{\bar{c}(t)}, m_j(t) \right) - \lambda z_j(t) \quad \tilde{\psi}_1 < 0, \quad \tilde{\psi}_2 < 0, \quad \lambda > 0
\]

(34)

The idea of pecuniary emulation involves the comparison of the level of consumption among individuals and therefore equation (34) is written at the level of individual workers explicitly. Given that there are \( N(t) \) workers, \( z_j(t) \) measures the strength of credit demand by worker \( j \) to keep up with her consumption standards. \( z_j(t) \) is properly scaled so as to yield an aggregate relation such that

\[
z(t) = \sum_{j=1}^{N(t)} z_j(t)
\]

is defined as the level of outstanding (real) debt of an individual worker \( j \) such that \( M(t)/p(t) = \sum_{j=1}^{N(t)} m_j(t) \). This notational convention applies to \( c_{w,j}(t) \) and \( c_{r,l}(t) \) as well: \( C_w(t) = \sum_{j=1}^{N(t)} c_{w,j}(t) \) and \( C_r(t) = \sum_{l=1}^{N(r)} c_{r,l}(t) \) with \( N^r(t) \) being the number of rentiers. \( \bar{c}(t) \) is the average consumption of rentiers, i.e.,

\[
\bar{c}(t) = \frac{C_r(t)}{N^r(t)}
\]

The emulation effect is captured by the negative effect of the relative consumption \( c_{w,j}(t)/\bar{c}(t) \) on the change in \( z_j(t) \), i.e., \( \tilde{\psi}_1 < 0 \). The lower the level of consumption of a worker relative to that of rentiers, the stronger her desire to emulate the rentiers, leading to a larger increase in her credit demand. In addition to the emulation effect, we include two other factors in the dynamics of \( z(t) \). First, a high debt position is likely to have a negative implication for the workers’ desire to take on more loans. The negative effect of \( m_j(t) \) on \( \dot{z}_j(t) \), \( \tilde{\psi}_2 < 0 \), captures the link. Second, the evolution of the workers’ desire to borrow to finance their consumption may be subject to their internal habits and therefore may contain an autoregressive component. The last term, \( -\lambda z_j(t) \), reflects this idea. \( \lambda \) is an inverse measure of the persistence of workers’ internal habits in borrowing decisions. Therefore the higher \( \lambda \), the more quickly the current level of workers’ desire to borrow loses its relevance.\(^{28}\)

---

\(^{28}\)The specification of (34) implies that the level of \( z_j(t) \) is determined by the following
Let us denote as \( \nu \) and \( \nu^r \) the number of workers and rentiers normalized by capital stock, respectively. Our analysis leave aside heterogeneity within each class. The symmetric treatment of workers and rentiers, respectively, leads to 
\[
\dot{z}(t) = \nu \dot{z}^j(t), \quad z(t) = \nu z^j(t), \quad m(t) = m^j(t) \nu, \quad c^w(t) = c^{w,j}(t) \nu \quad \text{and} \quad c^r(t) = \bar{c}(t) \nu^r.
\]
Using these relations, (34) can be written in an aggregate form:
\[
\dot{z}(t) = \tilde{\psi} \left( \frac{c^w(t)}{c - c^w(t)} \frac{\nu^r}{\nu}, \frac{m(t)}{\nu} \right) \nu - \lambda z(t) \quad (36)
\]
In this equation, \( \bar{c} \) is constant (see (24)) and \( \nu \) is also constant given our assumption that the utilization rate is at the desired rate and production technology is given by fixed coefficients.\(^{29}\) The evolution of the emulation motive also depends on the relative size of the social classes \( \nu^r/\nu \) but we do not know of any argument that the relative size of classes is crucial for the issues at hand and therefore take it as constant.\(^{30}\) Note that the right-hand side of (36) is decreasing in \( c^w(t) \) and \( m(t) \). Therefore we can simplify the notations by suppressing the constants from (36) and rewrite it as:
\[
\dot{z}(t) = \psi (c^w(t), m(t)) - \lambda z(t) \quad \psi_1 < 0, \quad \psi_2 < 0 \quad (37)
\]
Since the product market equilibrium determines \( c^w(t) \) as a function of \( m(t) \) and \( z(t) \), we have:
\[
\dot{z}(t) = \psi (c^w(m(t), z(t)), m(t)) - \lambda z(t) \equiv \Psi(m(t), z(t)) \quad (38)
\]
\[
\begin{align*}
\Psi_z &= \psi_1 c^w_z - \lambda \geq 0 \\
\Psi_m &= \psi_1 c^w_m + \psi_2 \geq 0
\end{align*}
\]
in integral:
\[
\int_0^t e^{-\lambda(t-\kappa)} \tilde{\psi} \left( \frac{c^{w,j}(\kappa)}{\bar{c}}, m^j(\kappa) \right) d\kappa \quad (35)
\]
The past trajectories of relative consumption and indebtedness enter the formation of \( z^j(t) \). Differentiating this equation with respect time yields eq\(^{30}\)
\[29\] \( \nu \) is constant because \( \nu = \frac{N(t)}{K(t)} = \frac{N(t)Y(t)}{Y(t)K(t)} = \frac{N(t)}{Y(t)} u^* \) and the Leontief technology implies \( \frac{N(t)}{Y(t)} \) is constant.
\[30\] Chapter 5 in Veblen (1912) discusses the possibility of endogenous fertility decisions: ‘the leisure class’ tends to restrain their desire to have offspring in order to keep the number of its membership from growing to dilute their prestigious status. It is not clear whether this consideration affects the relative size of the leisure class because such a motive to restrain self-reproduction may prevail in the rest of the society as well.
Eq. 38 is the fundamental equation in our model that governs the dynamics of the workers’ emulation motive. The negative effect of a rise in $z(t)$ on $c^w(t)$ tends to destabilize the emulation dynamics: an increase in the workers’ desire to keep up with rentiers shifts income distribution away from themselves and reduces their share of consumption, which encourages a further increase in their emulation motive. Such a destabilizing feedback, represented by $\psi_1 c^w_z$, is tamed by a stabilizing autoregressive force built in their internal habits, the effect captured by $-\lambda$. The net effect of $z(t)$ on $\dot{z}(t)$, therefore, is determined by the sign of $\Psi_z = \psi_1 c^w_z - \lambda$. The more sensitively workers respond to consumption inequality the larger $|\psi_1|$ and the more likely $\Psi_z$ is positive. The more persistent the movement of the workers’ desire to emulate rentiers the smaller $\lambda$ and the more likely $\Psi_z$ is positive.

The effect of $m(t)$ on $\dot{z}(t)$ is also ambiguous, i.e., $\Psi_m = \psi_1 c^w_m + \psi_2 \geq 0$. The first term $\psi_1 c^w_m$ is positive: an increase in the debt ratio adversely affects workers’ income and consumption, which stimulates the workers’ emulation motive. The workers’ desire to engage in the consumption race, however, is constrained by the level of their indebtedness, which is given by the negative term $\psi_2$.

6. Analysis

Eqs. 28 and 38 forms a recursive system. This section examines the properties of the debt-emulation dynamics.

6.1. Existence of steady state

A steady state $(m^*, z^*)$ of the system satisfies

$$\phi(y^w(m^*, z^*), z^*) = 0 \quad (41)$$
$$\psi(c^w(m^*, z^*), m^*) - \lambda z^* = 0 \quad (42)$$

Eq. 41 give a relation between $m$ and $z$ that bankers are content with and eq. 42 gives another relation that makes workers’ desire to emulate the rich stationary. Would a steady state exist that makes these two relations mutually consistent? We have not given specific forms to relevant functions and eqs. 41 and 42 can yield various steady state configurations including no, a unique or multiple steady state(s). The following assumptions are sufficient for the existence of a unique steady state.
Assumption 2. The $\psi$-function is bounded from above and below.

$$\underline{\psi} < \psi(c^w(t), m(t)) < \overline{\psi}$$  \hspace{1cm} (43)

Assumption 3. The negative effect of $m(t)$ on $\dot{z}(t)$, $|\psi_2|$, is sufficiently strong so that

$$|\psi_2|\tilde{m}' > |\psi_1||c^u_m\tilde{m}' + c^u_z| - \lambda$$ \hspace{1cm} (44)

Assumption 3 is likely to be satisfied for relatively large or small values of $z(t)$. If $z(t)$ is sufficiently large or small, the corresponding values of $\psi$ will be close to its lower or upper bounds around which $\psi$ is very flat with respect to $c^w(t)$. This means that $|\psi_1|$ is likely to be very small around high or small values of $z(t)$ and as a result (44) is satisfied. Assumption 3 asserts that the inequality holds for all intermediate values as well. This requires sufficiently high $|\psi_2|$ and $\lambda$. We will discuss the implication of the violation of this assumption at the end of section 6.2.

Proposition 2. (Existence of a unique steady state) There exists a unique steady state $(m^*, z^*)$ that satisfies eqs. 41 and 42.

Proof. see Appendix A.

6.2. Stability

The system of the debt and emulation dynamics has a useful property: any trajectories of $m(t)$ and $z(t)$ determined by eqs.28 and 38 are bounded.

Proposition 3. (Boundedness of trajectories) The system of eqs.28 and 38 produces bounded trajectories of $m(t)$ and $z(t)$.

Proof. See Appendix B.

Our key result regarding the stability of the system is given by the following proposition:

Proposition 4. (Existence of a stable limit cycle) Under the system of eqs.28 and 38 $(m(t), z(t))$ converges to a limit cycle if

$$\psi_1c^w_z - \lambda \equiv \Psi_z > |\Phi_m|$$ \hspace{1cm} (45)

or the stationary point $(m^*, z^*)$ if the inequality is reversed.
Proof. Under Assumptions 2 and 3, any trajectory of the system of eqs. 2 and 3 are bounded (Proposition 3) and there exists a unique stationary point (Proposition 2). These propositions satisfy the conditions required for the Poincare-Bendixson theorem (Hirsch and Smale, 1974). Thus a trajectory starting from an arbitrary initial point converges to either the unique stationary point or a limit cycle depending on whether or not the unique stationary point is locally stable. Assumption 3 ensures that \( \det(J) \) is positive and therefore the stability of the stationary point is determined by the sign of the trace.

\[
\text{tr}(J) = \Phi_m + \psi_1 c^w - \lambda
\]  

(46)

If \( \text{tr}(J) > 0 \), then the fixed point is unstable and there must exist a stable limit cycle. □

The trace condition for local instability has an important economic interpretation. It can be rewritten in terms of underlying behavioral parameters:

\[
|\psi_1|\phi_2 > \frac{\phi_1[slf_1r + f_2(1 + \alpha) + n + \lambda] + \lambda[1 - f_1(1 - s^f)]}{f_1(1 - s^f)}
\]  

(47)

Instability requires that \( \phi_2 \) is non-zero and sufficiently large but \( \phi_1 \) is relatively small. Thus the existence of bankers’ accommodating behavior (\( \phi_2 \neq 0 \)) is a precondition for instability and unstable scenarios require the degree of accommodation is sufficiently large. In addition, instability is more likely if there is only a weak link between the bankers’ credit supply and the worker’s net income (margin of safety), i.e., if \( \phi_1 \) is small.

The condition for instability is also related to the parameters that govern the emulation dynamics. Instability arises if the effect of consumption inequality on workers’ emulation behavior is strong (high \( |\psi_1| \)). The magnitude of \( |\psi_1| \) is related not only to workers’ psychology but also to social and cultural environments that make workers more status-conscious. Mass media, advertisements and social networks may play a particularly important role in this regard. Instability is also associated with a low value of \( \lambda \). This is not surprising because \( \lambda \), by definition, is an inverse measure of persistence or inertia in the habit formation and a low (high) value of \( \lambda \), therefore, makes the movement of \( z(t) \) more (less) persistent, exerting a destabilizing (stabilizing) force.

The system can be represented on the two-dimensional space. The nullcline for \( \dot{m}(t) \) has a positive slope (\( \dot{m}'(z(t)) > 0 \)). The shape of the nullcline
for $\dot{z}(t)$ is given by

$$\frac{\partial m}{\partial z} = -\frac{\Psi_z}{\Psi_m} = -\frac{\psi_1 c_z^w - \lambda}{\psi_1 c_m^w + \psi_2}$$

(48)

This expression can be either positive or negative, but given Assumption 3, the sign of (48) is positive in the neighborhood of an unstable steady state. Moreover, in such an unstable case, Assumption 3 ensures that the $z$-nullcline is flatter than the $m$-nullcline around the steady state. Figure 1 shows a phase diagram in the unstable case.

![Phase Diagram](image)

Figure 1: Phase Diagram

The phase diagram in the unstable case suggests that any trajectory starting from an arbitrary initial condition in the neighborhood of the fixed point exhibits a counterclockwise cyclical movement in the $(z(t), m(t))$ space. The trajectory spirals out as the fixed point is unstable and its boundedness implies that it eventually converges to a closed orbit. Figure 2 illustrates a

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31 If the steady state is unstable, the sign of $\text{tr}(J)$ is positive and therefore $\psi_1 c_z^w - \lambda$ must be positive around the steady state. If $\psi_1 c_z^w - \lambda > 0$, Assumption 3 is satisfied only if $\psi_1 c_m^w + \psi_2 < 0$. Thus (48) must be positive.
limit cycle that emerges from our system.\textsuperscript{32} The behavior of the debt ratio and the emulation motive has an affinity with the celebrated Volterra-Lotka predator-prey model, with \(m(t)\) and \(z(t)\) being the predator and the prey, respectively. At low debt ratios, the workers’ desire to emulate rentiers increases and, as bankers accommodate the workers’ credit demand, the debt ratio increases as well (region I). Sufficiently high debt ratios start to constrain the workers’ emulation motive \(z(t)\)\textsuperscript{33}, but the level of \(z(t)\) is still high and continues to accelerate debt accumulation (region II). The debt ratio eventually approaches its upper limit due to both supply- and demand-side factors in credit markets: (i) as increasing debt services associated with high debt ratios reduce the workers’ net income, bankers will be more reluctant to supply loans, and (ii) the decline in the workers’ credit demand due to their

\textsuperscript{32}Figures 2 - 4 are based on the following parameter values and specifications: \(u^* = 0.5\), \(\delta = 0.1\), \(n = 0.03\), \(r = 0.04\), \(s^f = 0.3\), \(\alpha = 1\), \(\lambda = 0.03\), \(c'(t) = 0.3y'(t) + 0.0305\omega'(t)\), \(\dot{m}(t) = 0.05(y^w(t) - 0.314) + 0.1z(t)\), \(\dot{z}(t) = \frac{0.06}{1 + \exp[0.04q(t)]} - 0.03 - \lambda z(t)\), and \(q(t) = 0.8925\frac{e^\gamma(t)/e^\epsilon(t)}{1 + e^\gamma(t)/e^\epsilon(t)} + 0.5m(t) - 0.9867\).

\textsuperscript{33}With \(m(t)\) and \(z(t)\) both being relatively high in region II, the workers’ consumption will be very low relative to rentiers, which tends to stimulate the workers’ emulation motive. The effect of high consumption inequality on the workers’ emulation motive is more than offset by the workers’ concerns over their high indebtedness. Therefore \(z(t)\) falls in region II.
excessively high debt positions results in a fall in actual borrowing. Thus the debt ratio starts to fall (region III). As the debt ratio becomes sufficiently low, the workers’ emulation motive starts to rise (region IV).  

Figure 3 depicts the cyclical movements of workers’ consumption and income. The vertical gap between these two series represents the amount of borrowing. Over the cycles, the workers’ income exhibits more substantial variations than their consumption. This is because of the effect of borrowing on consumption. Increasing borrowing augments consumption when income is falling.

\[ \frac{c^w(t)}{y^w(t)} \]

**Figure 3:** Workers’ income \((y^w(t))\) and consumption \((c^w(t))\)

Figure 4 demonstrates changes in consumption and income inequality, measured by the ratio of rentiers’ consumption and income to overall consumption and income, respectively. Inequality is more pronounced in income than in consumption. This is again due to the fact that workers borrow to compensate for large changes in income.

The violation of Assumption 3 opens a possibility of multiple equilibria. A low sensitivity of \(\dot{z}(t)\) to variations in \(m(t)\), for instance, may violate Assumption 3 for some values of \(z(t)\). If the inequality (44) is reversed at a steady state value of \(z(t)\), this steady state must be a saddle point and the

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34 This is simply a mirror image of what happens in region II that described in the previous footnote.
system contains at least two other steady states. Under some conditions, the economy may converge either to a ‘high’ or to a ‘low’ equilibrium depending on initial conditions. A high (low) level of indebtedness along with a strong (weak) emulation motive prevails in the high (low) equilibrium.

6.3. Different specifications

Our analytic results are based on the chosen specifications of the debt and emulation dynamics. In this section, we consider some alternative specifications and examine whether our results are robust to changes in the specifications.

6.3.1. Fast adjustment of the emulation motive

The emulation dynamics in section 5 treats $z(t)$ as a state variable: changes in $z(t)$ are associated with consumption inequality $(c^{w}(t)/c^{r}(t))$. One may instead take $z(t)$ as a fast variable so that the emulation effect works through the level of $z(t)$ rather than changes in $z(t)$. A simple case is given by

$$z(t) = \eta(c^{r}(t)), \quad \eta' > 0$$

One may find a similarity between this alternative approach and Dutt’s specification (Dutt, 2008).
which replaces eq. 36. Putting eq. 49 in eq. 15, we have:

\[ \dot{m}(t) = \phi(y^w(t), \eta(c^r(t))), \quad \phi_1 > 0, \quad \phi_2 > 0 \]  

(50)

The equilibrium condition for the goods market implies that both \( y^w(t) \) and \( c^r(t) \) are uniquely determined by \( m(t) \), \( y^w(t) \) is decreasing, and \( c^r(t) \) is increasing in \( m(t) \). Formally, denoting \( \Delta \equiv (1 + \phi_1) - (1 + \eta' \phi_2)(1 - s^f)f_1 > 0 \), we have

\[ \frac{\partial y^w(t)}{\partial m(t)} = -\frac{[rs^f f_1 + f_2(1 + \alpha)](1 + \eta' \phi_2) + n}{\Delta} < 0 \]  

(51)

\[ \frac{\partial c^r(t)}{\partial m(t)} = \frac{[rs^f f_1 + f_2(1 + \alpha)](1 + \phi_1) + (1 - s^f)f_1 n}{\Delta} > 0 \]  

(52)

These results tell us that the effect of \( y^w(t) \) on \( \dot{m}(t) \) – bankers’ emphasis on margins of safety – is conducive but that of \( c^r(t) \) – the effect of consumption inequality on the workers’ emulation motive – is detrimental to the stability of debt dynamics. Depending on the relative strength of these countervailing forces, the dynamics of workers’ indebtedness may or may not be stable. Straightforward algebra shows that the stability of the debt dynamics is determined by the sign of

\[ \frac{\partial \dot{m}(t)}{\partial m(t)} = \frac{[s^f f_1 r + f_2(1 + \alpha) + n(1 - s^f)f_1] \eta' \phi_2 - [s^f f_1 r + f_2(1 + \alpha) + n] \phi_1}{\Delta} \]

which yields the following stability condition.

\[ \frac{\partial \dot{m}(t)}{\partial m(t)} \geq 0 \iff \eta' \cdot \phi_2 \geq \phi_1 \left[ \frac{s^f f_1 r + f_2(1 + \alpha) + n}{s^f f_1 r + f_2(1 + \alpha) + n(1 - s^f)f_1} \right] \]  

(53)

This condition bears a resemblance to eq. 47 in its economic interpretation. It is easy to see that a sufficiently high value of \( \phi_2 \) (\( \phi_1 \)) makes the sign of \( \frac{\partial \dot{m}(t)}{\partial m(t)} \) positive (negative) and therefore the debt dynamics unstable (stable). The higher \( \eta' \) is, the more prone to instability the system is. While this uni-dimensional system does not yield cyclical behavior, the logic behind instability is not very different from our baseline model: the strength of the emulation motive and the nature of the financial system are critical for the properties of the debt dynamics.
6.3.2. Fully accommodating banks

Suppose that bankers fully accommodate workers’ demand for credit so that workers can consume any amount as they desire. Let us redefine $z(t)$ as the excess (or shortage) of workers’ desired consumption over their income, with the evolution of $z(t)$ still determined by our emulation dynamics, eq. 38.

By definition, we have

$$c^w(t) = y^w(t) + z(t)$$

and $z(t)$ can be either negative or positive. Plugging eq. 54 into the workers’ budget equation

$$\dot{m}(t) = c^w(t) - y^w(t) - nm(t)$$

we have:

$$\dot{m}(t) = z(t) - nm(t)$$

With (55) replacing (15), the qualitative properties of the system do not change much. (55) implies that $\dot{m}(t)$ still depends negatively on $m(t)$ and positively on $z(t)$. It can be also checked that workers’ consumption $c^w(t)$ is still affected negatively by both $m(t)$ and $z(t)$ under specification (55).\[36\]

The trace of the Jacobian matrix is given by $-n + \psi_1 z - \lambda$. Assuming that the determinant is positive, the steady state is unstable if

$$\psi_1 z - \lambda > n$$

i.e., if

$$|\psi_1| > (n + \lambda) \left[ \frac{1 - f_1(1 - s^f)}{f_1(1 - s^f)} \right]$$

The expression (57) is similar to (45) but $|\Phi_m|$ in (45) is replaced by $n$ in (57). The smaller $n$ and $\lambda$ and the larger $|\psi_1|$, the more likely the steady state is unstable.

7. Conclusions

The key element that makes the system unstable is the interaction between endogenous income distribution and pecuniary emulation. The state

\[36\] Denoting $\Delta_1 = 1 - f_1(1 - s^f)$, we have $\pi_m = f_1 r + f_2(1+\alpha) - r$ and $\pi_z = \frac{1}{\Delta_1 u^*} > 0$. The effect of $m(t)$ and $z(t)$ on consumption is given by $c^w_m = -\frac{f_1 r + f_2(1+\alpha)}{\Delta_1} < 0$ and $c^w_z = -\frac{f_1(1-s^f)}{\Delta_1} < 0$.\]
of unequal income distribution between workers and rentiers leads to consumption inequality, which triggers the workers’ desire to emulate rentiers. This raises the workers’ borrowing and stimulate aggregate demand. The Kaldorian adjustment mechanism implies that such an increase in aggregate demand shifts income distribution in favor of rentiers. The relative decline in the workers’ income share reduces the share of their consumption, which leads to a further increase in the workers’ desire to emulate the rich. Thus the workers’ effort to keep up with rentiers is frustrated, setting up an unstable process. A positive feedback mechanism of this kind arises because the state of income distribution is external to individual workers’ consumption decisions.

We have shown that the behavior of the system depends upon the banks’ lending policy. Workers use borrowing to attain their desired consumption but the actual amount of borrowing depends on the degree of banks’ accommodation of loan demand as well as banks’ concern over the workers’ cash flow position. Instability is more likely when the banks’ lending policy is more accommodating and less sensitive to variations in the workers’ net income. Thus the financial sector plays an essential role in determining the stability property of our model. Innovations in the financial sphere, along with changes in institutions and policies, in the run-up to the Great Reccession, appear to have made the banks’ lending policy more accommodating and less sensitive to the workers’ income position, thereby increasing the instability potential of the system.

Our analysis identifies a potential source of instability that emerges from consumption-debt dynamics, but it abstracts from a number of important factors. Our analysis, for instance, has paid no attention to heterogeneity within each groups and the emulation motive may be affected by the existence of within-group inequality. Another limitation is from our assumption that workers do not hold any asset such as housing. The interaction between asset prices and credit expansion may generate another important source of instability. Lastly, to the extent that our focus on the relation between income distribution and pecuniary emulation is empirically relevant, one may wonder what kind of policies would help stabilize the system. Our analysis

If workers took into account the negative external effect of their spending on income distribution and could coordinate their consumption plans, they would never choose to increase their spending.
points to the importance of policy alternatives that tackle the problem of consumption inequality. Addressing these issues, however, is left for future research.

References


Setterfield, M., 2013. Wages, demand and us macroeconomic travails: Diagnosis and prognosis, in: Cynamon, Barry, S.F., Setterfield, M. (Eds.),


Appendix A. Proof of Proposition 2

Assumption 2 ensures that the trajectory of $z(t)$ is bounded. To see this, note that Assumption 2 and equation (37) imply that

$$\psi < \dot{z}(s) + \lambda z(s) < \bar{\psi}$$

(58)

By integrating (58) over $[0, t]$ with respect to $s$, we have:

$$\exp(-\lambda t)z(0) + (1-\exp(-\lambda t))(\psi/\lambda) < z(t) < \exp(-\lambda t)z(0) + (1-\exp(-\lambda t))(\bar{\psi}/\lambda)$$

Since $0 < \exp(-\lambda t) \leq 1$, we obtain:

$$z_{\text{min}} \leq z(t) \leq z_{\text{max}}$$

(59)

where $z_{\text{min}} = \min\{z(0), \psi/\lambda\}$ and $z_{\text{max}} = \max\{z(0), \bar{\psi}/\lambda\}$.

Define $H(z(t)) \equiv \psi(c^u(\bar{m}(z(t)), z(t)), \bar{m}(z(t))) - \lambda z(t)$. By Assumption 2, we have:

$$H(z_{\text{min}}) = \psi(c^u(\bar{m}(z_{\text{min}}), z_{\text{min}}), \bar{m}(z_{\text{min}})) - \lambda z_{\text{min}} > \psi - \lambda z_{\text{min}} \geq 0$$

$$H(z_{\text{max}}) = \psi(c^u(\bar{m}(z_{\text{max}}), z_{\text{max}}), \bar{m}(z_{\text{max}})) - \lambda z_{\text{max}} < \psi - \lambda z_{\text{max}} \leq 0$$
Therefore $H(z_{\text{min}}) > 0 > H(z_{\text{max}})$. Assumption 3 implies that $H(z(t))$ is strictly decreasing in $z(t)$. Therefore the intermediate value theorem ensures the existence of a unique value $z^* \in (z_{\text{min}}, z_{\text{max}})$ such that $H(z^*) = 0$. The stationary value of $m$ is given by

$$m^* \equiv \tilde{m}(z^*)$$

**Appendix B. Proof of Proposition 3**

Since we have already proved in Appendix A that $z(t)$ is bounded, we only need to prove that $m(t)$ is bounded. Because $z_{\text{min}} \leq z(t) \leq z_{\text{max}}$, we have: $\tilde{m}(z_{\text{min}}) \leq \tilde{m}(z(t)) \leq \tilde{m}(z_{\text{max}})$. We claim: $m(t)$ is bounded such that

$$\min\{m(0), \tilde{m}(z_{\text{min}})\} \equiv m_{\text{min}} \leq m(t) \leq m_{\text{max}} \equiv \max\{m(0), \tilde{m}(z_{\text{max}})\}$$

To prove this, all we need to show is

(i) if $m(s) < \tilde{m}(z_{\text{min}})$, then $m(s)$ is increasing in $s$; if $m(s) > \tilde{m}(z_{\text{max}})$, then $m(s)$ is decreasing in $s$.

(ii) if $\tilde{m}(z_{\text{min}}) \leq m(s) \leq \tilde{m}(z_{\text{max}})$, then $\tilde{m}(z_{\text{min}}) \leq m(t) \leq \tilde{m}(z_{\text{max}})$ for all $t > s$

(i) can be easily shown: if $m(s) < \tilde{m}(z_{\text{min}})$, then $m(s) < \tilde{m}(z(s))$ and $\dot{m}(s) > 0$; if $m(s) > \tilde{m}(z_{\text{max}})$, then $m(s) > \tilde{m}(z(s))$ and $\dot{m}(s) < 0$. In other words, any trajectory that starts from a point outside the interval $[\tilde{m}(z_{\text{min}}), \tilde{m}(z_{\text{max}})]$ is attracted to this interval. Next, (ii) implies that any trajectory starting from a point in the interval $[\tilde{m}(z_{\text{min}}), \tilde{m}(z_{\text{max}})]$ cannot escape from this interval. To see this, suppose, without loss of generality, that there exists a $t' > s$ such that $m(t') < \tilde{m}(z_{\text{min}})$ and $\tilde{m}(z_{\text{min}}) \leq m(s) \leq \tilde{m}(z_{\text{max}})$. It follows from the continuity of the flow that there exists a $t''$ such that $s < t'' < t'$ and $m(t') < m(t'') < \tilde{m}(z_{\text{min}}) < m(s)$. By the mean value theorem, there exists a $t'''$ such that $t'' < t''' < t'$ and $\dot{m}(t''') = \frac{m(t'') - m(t')}{t''' - t'} < 0$. This, however, contradicts the fact that $\dot{m}(t)$ is positive whenever $m(t) < \tilde{m}(z_{\text{min}})$.

**Appendix C. Accounting relations**
Table 1: Balance Sheet Matrix

<table>
<thead>
<tr>
<th></th>
<th>Workers</th>
<th>Rentiers</th>
<th>Firms</th>
<th>Banks</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>0</td>
<td>0</td>
<td>$pK$</td>
<td>0</td>
<td>$pK$</td>
</tr>
<tr>
<td>Deposits</td>
<td>0</td>
<td>$M$</td>
<td>0</td>
<td>$-M$</td>
<td>0</td>
</tr>
<tr>
<td>Loans</td>
<td>$-M$</td>
<td>0</td>
<td>0</td>
<td>$M$</td>
<td>0</td>
</tr>
<tr>
<td>Equities</td>
<td>0</td>
<td>$vN$</td>
<td>$-vN$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sum</td>
<td>$-M$</td>
<td>$vN + M$</td>
<td>$pK - vN$</td>
<td>0</td>
<td>$pK$</td>
</tr>
</tbody>
</table>

Definitions. $v$: the price of equity, $N$: the number of equities.

Table 2: Transaction Flow Matrix

<table>
<thead>
<tr>
<th></th>
<th>Workers</th>
<th>Rentiers</th>
<th>Firms</th>
<th>Banks</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>$-pC^{w}$</td>
<td>$-pC^{r}$</td>
<td>$pC^{w} + pC^{r}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Investment</td>
<td>$W$</td>
<td>$W$</td>
<td>0</td>
<td>$-pI$</td>
<td>0</td>
</tr>
<tr>
<td>Wages</td>
<td>$W$</td>
<td>$W$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Firms’ profits</td>
<td>$Div$</td>
<td>$-(Div + \Pi^F)$</td>
<td>$\Pi^F$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Deposit interest</td>
<td>$iM$</td>
<td>0</td>
<td>$-iM$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Loan interest</td>
<td>$-iM$</td>
<td>0</td>
<td>$iM$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Change in wealth</td>
<td>$-(v\dot{N} + \dot{M})$</td>
<td>$v\dot{N}$</td>
<td>$\dot{M}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Change in loans</td>
<td>$\dot{M}$</td>
<td>0</td>
<td>0</td>
<td>$-M$</td>
<td>0</td>
</tr>
<tr>
<td>Sum</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Definitions. $\Pi^F$: retained earnings, $Div$: firms’ dividend payments to rentiers, $\dot{N}$: net issue of equities. The other variables are defined throughout the main text.