The Macrodynamics of Household Debt

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November 2010

Abstract
Recent research finds that corporate leverage affects macroeconomic dynamics and can contribute to financial fragility. We show that consumer debt is also important. We add consumer debt to a stock-flow consistent neo-Kaleckian growth model and explore the macrodynamic ramifications. Consumer debt influences effective demand, the profit rate, and economic growth. Unsurprisingly, laxer consumer credit constraints stimulate growth in the short run. However, the long-run effects may be growth reducing. Looser consumer credit can also make the system more vulnerable to changes in the state of confidence, the interest rate, and the saving propensity of rentiers. When consumer debt levels are high, a small increase in the interest rate or increase in the rentiers’s saving propensity, or reduction in the state of confidence can destabilize the macroeconomy.

We further extend the model endogenizing the retention ratio. We find that the model becomes structurally unstable. This allows a simple characterization of economic crisis: a downswing in the state of confidence destabilize the macroeconomy. We also observe that higher interest rates and more prudent behavior of rentiers can be destabilizing.

J.E.L. Codes: B59, E12, E22, O41
Keywords: Corporate debt, Consumer debt, Dynamics, Instability

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1 Introduction

Economists have long recognized that investment-finance behavior can influence business cycles. For example, Minsky's financial instability hypothesis proposes that debt dynamics contribute to macroeconomic instability (Minsky, 1980, 1986, 1992, 1995). The financial underpinnings of the recent recession has generated renewed attention to Minsky's finance-driven endogenous business cycle model. In this paper, we focus on a related factor that has been little emphasized in most Minskyan models: the role of household debt.

Minsky (1992) himself suggested that the validation of household debt (along with government and international debt) can contribute to business cycle dynamics.\(^1\) Nevertheless, household debt played little role in his work, and most subsequent research has also largely neglected household debt. Recent events suggest that this neglect is missing an important cause of macroeconomic fluctuations. From the 1980s to the 2000s, the US experienced consumption expansion accompanied by significant household debt accumulation. Cynamon and Fazzari (2008) argue that this provided a substantial macroeconomic stimulus. These levels of debt accumulation eventually proved untenable—a fact that has been broadly implicated in the recent recession. This provides a core motivation for our present research.

Naturally we are not seeking to overturn the stylized fact that investment is more volatile than GDP and consumption, nor the central role of investment in economic fluctuations. Rather, this project explores the additional contribution of household expenditures and debt to macroeconomic fluctuations.\(^2\) Certain stylized facts suggest this contribution may be growing in importance. For example, the ratio of personal outlays to disposable personal income increased from about 88 percent in the early 1980s to nearly 100 percent in 2007. Additionally, household debt outstanding as a share of GDP increased from about 45 percent in 1975 to nearly 100 percent in 2006. The magnitude of these shifts is startling and calls for research on their implications.

This paper is organized as follows. Section 2 reviews some relevant literature. Sections 3–6 present and analyze a growth model with both consumer and corporate debt. Section 7 concludes.\(^3\)

2 Business Cycles and Household Debt

In this section we examine the business cycle behavior of two financial series derived from the Flow of Funds Accounts of the United States. (The data are not seasonally adjusted.) We deflate the nominal data with the personal consumption chain-type price index from the national income and product accounts.

The NBER provides reference dates for business cycles at \url{http://www.nber.org/cycles/cyclesmain.html}. Due to data availability, we consider business cycles occurring between

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\(^1\)The term "validate" is often used by Minsky to describe the process of meeting contractual debt service obligations (Minsky, 1986, 1992).

\(^2\)Hall (1986) provides an important early empirical study on the contribution of consumption behavior to economic fluctuations.

\(^3\)Related econometric analysis is presented in a separated paper (Kim, 2010).
1951 and 2000. Figure 1 displays the corresponding levels of real household debt and net worth over the cycle. Here household debt combines consumer and mortgage debt. We see a tendency for the level of household debt to rise over the history of US business cycles.

![Graph of Household Debt](image1)
![Graph of Household Net Worth](image2)

Data Source: Flow of Funds Accounts

Figure 1: Real household debt and net worth over business cycles

Figure 1 also reveals higher volatility in household net worth beginning in the later periods of the last full business cycle (1991Q2-2001Q4). We also observe that the length of the business cycles appears longer in the more recent cycles.

We observe interesting differences between earlier and later cycles. We distinguish between cycles before and after the early 1980s. (Cycles over the following periods are not presented, as they are very short: 1951Q4-1954Q2, 1958Q3-1961Q1, and 1980Q4-1982Q4.) In the pre-1980s business cycles, the growth rate of household debt shows a downward trend, reaching a negative range over a number of periods. In later cycles, the growth rate of household debt is mostly positive, and we observe an upward trend in the growth rate over the business cycle of 1991Q2-2001Q4 (see the figure 2). Overall there are clear differences between later and earlier cycles. In earlier cycles, there is a visible downward trend of all components of household debt. On the other hand, we observe an upward trend in all of the components of household debt for the business cycle period of 1991Q2-2001Q4. For later business cycles, we also generally observe a positive growth rate for mortgage and household debt—essentially from 1983Q1 to the start of the current deep recession.

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4So our data for the 1949–1954 business cycle begins a several quarters into the recession.
5The increase in volatility of household net worth is even clearer when we look at its growth rate. The graph is not presented in this paper for reason of space.
6The graphs for the growth rates of mortgage debt and consumer debt show similar patterns. They are not presented in this paper for reason of space.
3 Literature Review

Research on the impact of debt on business cycles and economic growth usually emphasizes the role of corporate debt (Skott, 1994; Taylor and O’Connell, 1985; Lavoie, 1995; Foley, 2003; Hein, 2006; Lima and Meirelles, 2006, 2007). The majority of these studies invoke Minsky’s financial instability hypothesis as a conceptual foundation. The financial instability hypothesis relates debt, particularly corporate debt, to systemic financial fragility.

Minsky distinguishes three different financial states of firms: hedge, speculative, and Ponzi. Firms that can cover all their debt service obligations with their cash flow are in the “hedge” state. Firms move to the “speculative” state when they can pay interest payments out of their cash flow but cannot cover the repayment of the principal. They move the ‘Ponzi’ state when cash flow is not sufficient even to cover the interest payments on the firm’s extant debt. Minsky argues that a prolonged period of prosperity will induce euphoric expectations, leading firms to adopt riskier and riskier financial stances. As the average firm evolves from hedge to largely speculative and even Ponzi finance, the economy becomes systemically fragile and susceptible to a sudden financial crisis.

Hein (2006) builds upon the work of Lavoie (1995) to incorporate interest payments and corporate debt into a neo-Kaleckian growth model. (See section 4 for a discussion of the neo-Kaleckian approach.) He finds a possible short-run positive relationship between accumulation and the real interest rate. This is Lavoie’s “puzzling case”.\(^7\) Hein also studies the long-run effect of interest rate changes on economic growth and macro stability under different conditions.

Charles (2008a,b) explores the emergence of financial instability in a neo-Kaleckian growth model with corporate debt and debt-financed investment. Whereas Charles (2008a) endogenizes the retention ratio of the firm, Charles (2008b) links the financial structure as defined by Minsky to capital accumulation. In both studies, a high interest rate emerges as

\(^{7}\)We will attribute this to the accelerator effect in his investment function (section 6.1).
a likely precondition to financial instability.

Palley (1994) and Dutt (2006) are exceptional in that they investigate the role of consumer debt in the business cycle. Palley (1994) uses a linear multiplier-accelerator model to analyze cyclical aspects of household borrowing over the business cycle: a rise in consumer debt initially increases consumption and hence promotes growth, but eventually the accumulation of debt becomes excessive. The debt service burden then reduces consumption and the output level. Dutt (2006) investigates the role of consumer debt within a neo-Kaleckian distribution framework, finding different results from Palley (1994). In the model of Dutt (2006), an increase in household consumption debt raises the growth rate in the short run. In the long run, however, the effect is ambiguous because the accumulation in consumer debt results in a shift in income distribution toward rentiers, who have a higher propensity to save. This latter result has depressing effect on the long-run growth rate. Cynamon and Fazzari (2008) provide an informal but informative discussion of household debt from a Minskyan perspective. They suggest that social factors, such as changes in consumption and borrowing norms, are important for the rise in consumer and household debt in the US. They also suggest that the unprecedented rise in household debt could have planted the seeds for financial instability and a nontrivial economic downturn.

4 Basic Modeling Framework

In this section we lay out our accounting framework and present the structure of an initial theoretical model. This initial model is an extension of extant neo-Kaleckian literature on debt dynamics, and embodies the strong assumption that firms retain all net profits after paying interest on their debt (so there is no equity market and there are no dividend payments). As will be seen below, this strong assumption leads to certain implausible results, which will be addressed by introducing equity and dividend payments in section 7. The simpler model is presented first, however, because it more clearly reveals the logic of the modeling approach and is easier to explain intuitively, and because most of the comparative static and comparative dynamic results from the simpler model are found to be robust in the more complex model with equity and dividends (although the latter model yields a more plausible range of possible steady-state solutions).

Before laying out the model, we briefly address our choice of modeling framework. Neo-Kaleckian model of growth and distribution was originally proposed by Del Monte (1975), Dutt (1984) and Rowthorn (1982), among others. Blecker (2002) provides a survey and discussion of later extensions. Neo-Kaleckian macromodels emphasize demand-driven output and growth, the importance of profitability in the accumulation process, and the importance of income distribution for macroeconomic outcomes. Existing neo-Kaleckian models have examined the role of corporate debt and, to a lesser extent, household debt (Charles, 2006b,a; Dutt, 2005, 2006). They therefore provide natural reference points and building blocks for our formal model. Additionally, Kalecki's economics provides fundamental background for Minsky's financial instability hypothesis, and Minsky's hypothesis provides a useful conceptual framework for us.

Kalecki's profit relation and principle of increasing risk are cornerstones of Minsky's financial instability hypothesis. Kalecki's profit relation establishes that effective demand
determines aggregate profits, and Minsky finds in this significant implications for macroeconomic instability. For example, profits increase during an investment boom, which validates optimistic expectations and thereby contributes to a speculative investment behavior that creates instability in the system. Kalecki's principle of increasing risk provides a foundation for Minsky's twin concepts of "lender's and borrower's risk" in his two-price system, which is a basic building block of the micro-foundation for the financial instability hypothesis.

4.1 Social Accounting Matrices

We begin the model by laying out a basic accounting framework, which closely follows Lavoie and Godley (2002). We distinguish three types of agents in this model, which we call workers, rentiers, and firms. To focus the model on the role of consumption behavior, we consider a closed economy with no government contribution to aggregate demand. Notation is defined in the text and summarized in table B.1 of the notation appendix.

Table 1 is the balance sheet matrix for our model economy. It shows the asset and liability allocations across our three types of agent. We initially consider three types of assets: physical capital ($K$), loans to households ($D_W$), and loans to firms ($D_F$). We do not introduce a separate housing sector, so our analysis will focus on consumer debt (i.e., we do not consider mortgage debt or home equity). The column sums (across capital and loans) produce the net worth of each class of agent, while the row sums (across workers, rentiers, and firms) produce the net value of each class of assets. Note that we do not assume that the measured net worth of firms is zero.

<table>
<thead>
<tr>
<th>Table 1: Balance sheet matrix</th>
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<tbody>
<tr>
<td>Workers</td>
</tr>
<tr>
<td>Capital</td>
</tr>
<tr>
<td>Loans</td>
</tr>
<tr>
<td>Net worth</td>
</tr>
</tbody>
</table>

Associated with this balance sheet matrix is the transaction flow matrix in table 2. In the case of households, their real wage income ($W_rL$) can be supplemented by new borrowing ($D_W$) to finance consumption ($C_W$) and interest on past borrowing ($iD_W$). Rentiers have income from the loans they have made ($iD_W + iD_F$), which they use for consumption ($C_R$) or to make new loans ($D_W + D_F$). In the case of firms, we distinguish between capital and current transactions. Firms can finance investment ($I$) with new borrowing ($D_F$) or retained earnings ($\Pi_F$). In the initial specification of the model, there is no publicly held equity. In this sense, capitalists are identical to firms. Related to this, all of the firms' earnings

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Minsky distinguishes between the price systems for current output and capital assets. Once external finance is involved, the supply price of capital assets differs from the current output price due to external financing cost, which is referred as "lender's risk". Similarly, once the external borrowing for purchasing the capital assets is involved, the demand price of capital asset must include "borrower's risk" since greater borrowing exposes the buyer to higher risk of insolvency. For more detailed discussion, see Minsky (1986, pp. 194-218). Also see Papadimitriou and Wray (2008) and Wray and Tympoigne (2008).
after debt service payment are retained. For simplicity, a common interest rate \(i\) applies to consumer debt and corporate debt. For the transaction matrix, we note that the sums across the rows must equal zero as a consistency condition. The columns also sum to zero reflecting budget constraints.

<table>
<thead>
<tr>
<th>Table 2: Transaction Flow Matrix</th>
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<tbody>
<tr>
<td></td>
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<tr>
<td>Consumption</td>
</tr>
<tr>
<td>Investment</td>
</tr>
<tr>
<td>Wages</td>
</tr>
<tr>
<td>Firms' profits</td>
</tr>
<tr>
<td>Loan interest</td>
</tr>
<tr>
<td>Change in loans</td>
</tr>
<tr>
<td>Sum</td>
</tr>
</tbody>
</table>

4.2 Firms

Firms are characterized by their investment demand behavior and markup pricing behavior. In this version of the model, firms pay no dividends, so gross profit (\(\Pi\)) is split between retained earnings and debt service.

\[
\Pi = \Pi_F + iD_F
\]

(1)

Letting \(r = \Pi/K\) denote the gross profit rate, \(r_F = \Pi_F/K\) denote the retained earnings rate, and \(d_F = D_F/K\) denote the leverage ratio (corporate debt/capital), we can correspondingly decompose the gross profit rate into the sum of the retained earnings rate and the leverage ratio.

\[
r = r_F + id_F
\]

(2)

The retained earnings rate plays a central role in our characterization of investment demand. Many empirical studies find retained earnings to be an important determinant of investment (Fazzari and Mott, 1986-1987; Fazzari et al., 1988; Chirinko and Schaller, 1995; Ndikumana, 1999; Chirinko et al., 1999). As in Jarsulic (1996) and Charles (2008a), we therefore model investment demand as a response to retained earnings: the desired investment rate \(g_K = I/K\) responds to the retained earnings rate \(r_F\).

\[
g_K = \kappa_0 + \kappa_r r_F
\]

(3)

The parameters are positive: \(\kappa_0\) captures the level of “animal spirits” or the state of business confidence, and \(\kappa_r\) captures the sensitivity of desired investment to the retained earning rate. In addition, \(\kappa_r < 1\). (See footnote 12.)

\(^9\) In a future extension of the model, equity will be introduced, and capitalists will be included in addition to rentiers.
We treat the pricing behavior of firms in standard neo-Kaleckian fashion: price is a markup over unit labor costs, reflecting an oligopolistic market structure (Harris, 1974; Asimakopulos, 1975).

\[ P = (1 + \tau)W_nL/Y \]  

(4)

Here \( P \) is the price level, \( W_n \) is the nominal wage, \( \tau \) is the constant markup rate (which represents Kalecki's degree of monopoly), and \( L/Y \) is the labor-output ratio (i.e., the inverse of labor productivity). Such markup pricing behavior implies a standard expression for the gross profit share (\( \pi = \Pi/Y \)):

\[ \pi = \frac{\tau}{1 + \tau} \]  

(5)

Since the profit share is determined directly by the markup, our exposition below will generally treat \( \pi \) as a model parameter.

4.3 Workers

As indicated by the social accounting matrix in table 2, workers can borrow to raise their consumption above their current income. Correspondingly, the consumer may need to pay interest on outstanding household debt. The traditional neo-Kaleckian consumption function, in which workers consume all their wage income each period, does not allow for these considerations. To allow for debt financed consumption, we offer two modifications of the aggregate consumption function: we explicitly account for the payment of interest on household debt, and we include a term that captures the influence of credit conditions on consumer spending.

\[ C_W = W_rL - iD_W + \theta(D_{\overline{W}} - D_W) \]  

(6)

The formulation in (6) encompasses models that constrain consumption to equal current wages. (Just set \( D_W = 0 \) and \( \theta = 0 \).) The term \( W_rL - iD_W \) is after-interest disposable income. Here \( \theta > 0 \) is an adjustment coefficient, and the "credit target" \( D_{\overline{W}} \) summarizes current credit conditions in the macroeconomy. From (5) we know the share of real wages in income is \( 1 - \pi = 1/(1 + \tau) \), we we can also write (6) as

\[ C_W = (1 - \pi)Y - iD_W + \theta(D_{\overline{W}} - D_W) \]  

(7)

Our use of the term 'credit conditions' is rather broad: it is intended to summarize the financial practices of both lenders and borrowers, as influenced by institutional and cultural norms.\(^{10}\)

To learn the implications of our formulation of consumption behavior, recall from table 2 that

\[ \dot{D}_W = C_W + iD_W - W_rL \]  

(8)

\(^{10}\)See Cynamon and Fazzari (2008) for further study on the institutional and cultural influences on changes in financial practices. Our formation is admittedly an ad hoc way of representing the institutional and cultural norms of the financial practices. A valuable extension of the current work would be endogenizing this parameter reflecting the changes in the economic environment.
(This is just a budget constraint.) Therefore the consumption behavior in (6) implies that

\[ \dot{D}_W = \theta(\bar{D}_W - D_W). \]  

(9)

Note that \( \bar{D}_W \) regulates contemporary credit flows but not the outstanding stock of credit: the latter is determined historically. When there is an increase in \( \bar{D}_W \), consumption spending increases. When current borrowing falls below the credit target \( \bar{D}_W \), consumers can borrow so that consumption can exceed after-interest disposable income.\textsuperscript{11}

### 4.3.1 Rentiers

Rentiers consume out of their interest income,

\[ C_R = (1 - s_R)(iD_W + iD_F) \]  

where \( s_R \) is the rentiers' saving coefficient. (Equivalently, rentiers' saving is \( S_R = s_R(iD_W + iD_F). \) Recall from the rentiers' column in transaction flow matrix (table 2) that

\[ C_R = (iD_W + iD_F) - (\dot{D}_F + \dot{D}_W) \]  

(11)

Equations (10) and (11) imply the following stock/flow linkage:

\[ \dot{D}_F + \dot{D}_W = s_R(iD_W + iD_F) \]  

(12)

It follows that the saving behavior of rentiers is a key determinant of the growth rate of gross indebtedness in the economy.

\[ \frac{\dot{D}_F + \dot{D}_W}{D_W + D_F} = s_Ri \]  

(13)

Note that from a portfolio perspective the model has effectively one asset, since rentiers treat consumer debt and corporate debt as perfect substitutes.

### 5 Temporary Equilibrium

Commodity market equilibrium in this model has a standard representation:

\[ Y = C_W + C_R + I \]  

(14)

Substituting from the consumption equations (7) and (10):

\[ Y = [(1 - \pi)Y - iD_W + \theta(\bar{D}_W - D_W)] + [(1 - s_R)(iD_W + iD_F)] + I \]  

(15)

\textsuperscript{11}We are inclined to find some support for this macroeconomic story in the recent microeconomic empirical evidence of Gross and Souleles (2002). Using US data on a panel of thousands of individual credit card accounts, they found that consumers increased their credit card debt following increases in their credit limits.
Normalizing all variables by the capital stock produces

$$\frac{Y}{K} = (1 - \pi) \frac{Y}{K} - \frac{dW}{K} + \theta \frac{dW}{K} - dW + (1 - s_R) \frac{dW}{K} + \frac{I}{K} \tag{16}$$

Let $u = Y/K$ denote capacity utilization, $dW = dW/K$ denote the normalized indebtedness of workers, and $\bar{dW} = \frac{dW}{K}$ denote our credit target. Then

$$u = (1 - \pi)u - idW + \theta(dW - dW) + (1 - s_R)\theta(dW + dF) + g_K \tag{17}$$

Substituting from the investment demand equation (22), we finally obtain our preferred representation of commodity market equilibrium.

$$u = \theta(dW - dW) + (1 - \pi)u - idW + (1 - s_R)i(dW + dF) + [\kappa_0 + \kappa_r(\pi u - idF)] \tag{18}$$

Solving (18) for capacity utilization, we obtain

$$u = \frac{1}{\pi(1 - \kappa_r)}[\theta(dW - dW) - s_R idW + (1 - s_R - \kappa_r) idF + \kappa_0] \tag{19}$$

The profit rate, the retained earnings rate, and the accumulation rate can be expressed in terms of capacity utilization rate. The gross profit rate is just the product of the gross profit share and the capacity utilization rate—a relationship that allows us to reduce the expression for the retained earnings rate and thereby the accumulation rate.

$$r = \pi u \tag{20}$$

$$r_F = \pi u - idF \tag{21}$$

$$g_K = \kappa_0 + \kappa_r(\pi u - idF) \tag{22}$$

Substituting from (19) we therefore have the following solutions for the profit rate, the retained earnings rate, and the accumulation rate.

$$r = \frac{1}{(1 - \kappa_r)}[\theta(dW - dW) - s_R idW + (1 - s_R - \kappa_r) idF + \kappa_0] \tag{23}$$

$$r_F = \frac{1}{(1 - \kappa_r)}[\theta(dW - dW) - s_R i(dW + dF) + \kappa_0] \tag{24}$$

$$g_K = \frac{1}{(1 - \kappa_r)}[\kappa_0 + \kappa_r(\theta(dW - dW) - \kappa_r s_R i(dW + dF))] \tag{25}$$

5.1 Comparative Statics

This section presents the comparative statics analysis of temporary equilibrium. When possible, we sign the response of $u$, $r$, and $g_K$ to changes in the model parameters. Table 3 summarizes the results.\(^{12}\)

\(^{12}\) These results impose $\kappa_r < 1$. Neo-Kaleckian models often justify this kind of restriction on the marginal propensity to spend as a short run stability condition. To see why, base very short term output adjustment (a) on the excess demand function, defined by $ed = (C + I - Y)/K$. Examining $d\bar{u}/du$ near an equilibrium,
Table 3: Short-Run Comparative Statics ($u$, $r$, and $g_K$)

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$r$</th>
<th>$g_K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_0$</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$i$</td>
<td>?</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>$d_F$</td>
<td>?</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>$d_W$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{d}_W$</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$s_R$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\pi$</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Consider first the effect on an increase in $i$. This affects the economy through a consumer debt channel and a corporate debt channel. Consider first the consumer debt channel. A higher interest rate causes a reduction in workers’ after-interest disposable income, for an positive level of consumer debt. However it correspondingly increases rentiers’ income. The net effect is lower effective demand, since workers have a higher marginal propensity to consume than rentiers.

Consider next the corporate debt channel. An increase in the interest rate causes a reduction in retained earnings for any positive level of corporate debt, and hence a fall in investment demand. These demand reductions tend to reduce $u$. On the other hand, there is a corresponding increase in rentiers’ consumption, since they are now collecting more interest income from firms. Generally, the net effect is ambiguous, as indicated in table 3.

However, if rentiers have a high savings rate, the overall effect is negative. Looking at equation (19), we see that the ambiguity arises from the term $(1 - s_R - \kappa_r)$. Although both $\kappa_r$ and $s_R$ are less than one, their sum may well exceed 1 without violating the stability condition $\kappa_r < 1$. Indeed, a key assumption of Charles (2008a) is that $(1 - s_R - \kappa_r) < 0$, which ensures a negative impact of interest rate increases on $u$ and $r$.

Even without the rentier-saving assumption, an increase in the interest rate unambiguously reduces the accumulation rate. This is because it unambiguously reduces retained earnings: The direct response of retained earnings ($-d_F$) necessarily outweighs the indirect effect via the profit rate.\(^\text{13}\)

Similar considerations apply to the impact effect of change in corporate debt. Higher corporate debt results in higher interest payment for firms, cutting retained earnings and reducing accumulation. At the same time, it also results in higher interest income for rentiers, we obtain $\kappa_r < 1$ as a stability condition. This condition ensures that investment responds less strongly than aggregate saving to an increase in capacity utilization. Ceteris paribus, an additional unit of income will generate $(1 - \pi)$ units of consumption and $\pi$ units of retained earnings, of which $\kappa_r, \pi$ will be used for new investment. Unless $\kappa_r < 1$, a rise in income will increase excess demand.

\(^\text{13}\)Suppose we reverse Charles's rentier-saving assumption and assume that $(1 - s_R - \kappa_r) > 0$. This would give us a capacity utilization rate that responds positively to an increase in interest rate, similar to the result obtained by Hein (2006). In contrast to Hein's model, however, our present model does not suggest any possibility of a positive response of accumulation rate to an increase in interest rate. (This is the “puzzling” case of Lavoia (1995).) To produce that, we would have to additionally incorporate an accelerator effect in our investment function (e.g., a positive response of the accumulation rate to $u$). Our model then could exhibit the ‘puzzling’ case of positive relationship between interest and accumulation rates.
leading to higher consumption. The overall impact on \( u \) and \( r \) is therefore ambiguous. Taylor (2004) suggests that we distinguish between debt-led and debt-burdened effective demand, which depends on the relative magnitudes of these effects. Here we find it necessary to draw somewhat finer distinctions. In general, higher corporate debt has an ambiguous effect on effective demand. Under the rentier-saving assumption, however, the effect is definitely negative. Furthermore, even without the rentier-saving assumption, great corporate debt suppresses growth by suppressing the accumulation rate. In this sense growth is debt-burdened.

The effects of consumer indebtedness are somewhat easier to trace. An increase in the credit target relative to the capital stock \( d_W \) allows consumer to increase their consumption spending. The resulting increase in effective demand increases capacity utilization, which in turn increase the profit rate and thereby the rate of accumulation. Unsurprisingly then, an increase in the level of consumer indebtedness \( (d_W) \) has the opposite effects. Higher consumer debt implies higher interest payments for workers and more income for rentiers. Since workers have the higher propensity to consume, this reduces total consumption. The result is lower capacity utilization, retained earnings, and investment.

Finally, an increase in the profit share \( (\pi) \) redistributes income away from workers, reducing consumption demand. This decline in effective demand reduces capacity utilization in proportion. As in Charles (2008a), our neo-Kaleckian model of debt dynamics is wage-led or "stagnationist".

6 Dynamics

In this section we analyze the dynamics of our model. The dynamics are somewhat complex, so we add intuition by first considering two special cases.

6.1 The Case of \( d_W = 0 \)

Prior to the dynamic analysis of the full model, we consider the model in the absence of consumer debt. This will serve two purposes. First, the model without consumer debt is more easily compared to extant neo-Kaleckian literature on debt dynamics, which focuses primarily on the role of corporate debt (Lavoie, 1995; Hein, 2006; Charles, 2008a,b). Second, the restricted dynamics provide a point of comparison for the dynamics of the full model. (In the next section, the model without corporate debt will be analyzed.) Comparison with the full model will demonstrate that the addition of consumer debt can alter steady state outcomes and macroeconomic dynamics. In the absence of consumer debt, the consumption functions for workers and rentiers become

\[
C_W = W_nL \tag{26}
\]

\[
C_R = (1 - s_R)iD_F \tag{27}
\]

The rest of the model is unchanged. The temporary equilibrium value of capacity utilization therefore simplifies to

\[
u = \frac{1}{\pi(1 - \kappa_r)}[(1 - s_R - \kappa_r)iD_F + \kappa_0] \tag{28}
\]
The implied values for $r$, $r_F$, and $g_K$ are

$$ r = \frac{1}{(1 - \kappa_r)}[(1 - s_R - \kappa_r)i d_F + \kappa_0] $$

$$ r_F = \frac{1}{1 - \kappa_r}(-is_Rd_F + \kappa_0) $$

$$ g_K = \frac{1}{1 - \kappa_r}(\kappa_0 - \kappa_r s_R i d_F) $$

The comparative statistic results in table 3 are preserved in this simplified case (see table 4).

<table>
<thead>
<tr>
<th>$u$</th>
<th>$r$</th>
<th>$g_K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_0$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$i$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$d_F$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$\pi$</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>$s_R$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4: Short-Run Comparative Statics without Consumer Debt ($u$, $r$, and $g_K$)

Next we look at the model dynamics, which can be summarized in the behavior of $d_F$. From the transactions flows in Table 2, $\bar{D}_F = I - \Pi_F$, so we have

$$ \dot{d}_F = \frac{\bar{D}_F}{K} - \frac{K \bar{D}_F}{K^2} $$

$$ = \frac{I - \Pi_F}{K} - \frac{I D_F}{K K} $$

$$ = g_K - r_F - g_K d_F $$

From (31) and (30) we see that

$$ g_K - r_F = s_R i d_F $$

Therefore

$$ \dot{d}_F = (s_R i - g_K)d_F $$

This suggests two steady states. The first ($d_{F1}$) involves the natural steady-state condition

$$ s_R i = g_K $$

which just says that in a steady-state debt must grow at the same rate as the capital stock (recall equation 13). In the second steady state ($d_{F2}$), investment is financed purely by internal funds, so $d_{F2} = 0$.

Let us consider the first of these steady states for a moment:

$$ s_R i = \frac{\kappa_0 - \kappa_r s_R i d_{F1}}{1 - \kappa_r} $$

13
Solving for \(d_{F1}\) we get
\[
d_{F1} = \frac{\kappa_0 - (1 - \kappa_r)s_i}{\kappa_r s_i}
\] (37)

Thus \(d_{F1}\) could be either positive or negative depending on the relative magnitude of \(\kappa_0\) and \((1 - \kappa_r)s_i\). If animal spirits \((\kappa_0)\) were low enough, \(d_{F1}\) would be negative, which implies that firms would be rentiers at this steady state. However, since \(s_i\) is small (both components are fractions) this case has little relevance. We assume animal spirits are high enough to ensure \(d_{F1}\) will be positive, implying that the capital stock is partly debt financed in this steady state.

It is easy to see why the positive steady-state \(d_{F1}\) is unstable. Consider a steady state at \(d_F > 0\), where the saving of rentiers is just adequate to meet the demand to debt finance the capital stock. Now reduce \(d_F\) slightly (e.g., by reducing \(D_F\)). This reduces the debt finance requirements of firms, which then have more internal funds available for investment. The capital stock grows more quickly in response, driving \(d_F\) lower yet.

Now let us consider the second of these steady states for a moment. Note that
\[
\frac{dd_F}{dd_F} \bigg|_{d_F=0} = s_i - g_K
\]
\[
= s_i - \frac{\kappa_0}{1 - \kappa_r}
\]
\[
= \frac{(1 - \kappa_r)s_i - \kappa_0}{1 - \kappa_r}
\]
\[
= \frac{\kappa_r s_i}{1 - \kappa_r} d_{F1}
\] (38)

That is, \(d_{F2} = 0\) is stable steady-state when the other (unstable) steady state is at a positive level of corporate debt.

Next we give a somewhat more detailed algebraic treatment of the same points:
\[
d_F = \frac{\dot{D}_F}{K} - \frac{KD_F}{K^2}
\]
\[
= s_i d_F - g_K d_F
\]
\[
= [s_i - \frac{1}{1 - \kappa_r} (\kappa_0 - \kappa_r s_i d_F)] d_F
\] (39)
\[
= \frac{1}{1 - \kappa_r} [(1 - \kappa_r)s_i - (\kappa_0 - \kappa_r s_i d_F)] d_F
\]
\[
= \frac{1}{1 - \kappa_r} [-\kappa_0 d_F + (1 - \kappa_r)s_i d_F + \kappa_r s_i d_F^2]
\]

Clearly \(d_{F2} = 0\) is a steady state. Factoring this out determines another steady state to be \(d_{F1} = [\kappa_0 - (1 - \kappa_r) s_i]/\kappa_r s_i\). We explore stability by examining the slope of \(d_F\)
Figure 3: Univariate Dynamics of Corporate Debt

evaluated at these steady states.

\[
\frac{dd_F}{dd_F} = \frac{1}{(1 - \kappa_r)}\{-\kappa_0 + (1 - \kappa_r)i s_R + 2\kappa_r i s_R d_F\}
\]

\[
= \frac{-\kappa_r s_R}{(1 - \kappa_r)} \left\{ \frac{\kappa_0 - (1 - \kappa_r) s_R i}{\kappa_r s_R i - 2d_F} \right\}
\]

\[
= \frac{-\kappa_r s_R}{(1 - \kappa_r)} \{d_{F1} - 2d_F\}
\]

So stability is contingent. If \(\kappa_0 - (1 - \kappa_r)i s_R > 0\), then \(d_{F1} > 0\) is an unstable steady state, but the steady-state point \(d_{F2} = 0\) is stable (i.e., \(dd_F/dd_F < 0\)). Figure 3 illustrates this case in an univariate phase diagram. On the other hand, if \(\kappa_0 - (1 - \kappa_r)i s_R < 0\), we find \(d_{F1} < 0\) is a stable steady state, and the steady-state point \(d_{F2} = 0\) is unstable. For the remainder of this paper, we focus on the first case which is the one illustrated in 3. That is, we assume that \(\kappa_0 - (1 - \kappa_r)i s_R > 0\), since \(\kappa_r, s_R,\) and \(i\) are fractions, which makes the term \((1 - \kappa_r)i s_R\) very small.

Our simplified model in this section essentially has a similar structure to the model of Hein (2006) except for the presence of accelerator effect in his investment function. However, the result is quite contrastable. Our model, in the static case, does not show any possibility of a positive comparative static relationship between interest and accumulation rates (Hein's "puzzling case"). This different result is solely due to the absence of accelerator effect in our model. In the dynamics, our model cannot possess a stable steady state at a positive debt-capital ratio, while a stable steady state at a positive level is a possibility and a focal point of the analysis in Hein's study.\(^1\)

\(^1\)The discussion in Hein (2006) is incomplete. Although his model has a steady state at zero leverage level, there is no reference to it. Similar to our model, his model could have a steady state at a negative leverage,
Our result in this section is also comparable to Charles (2008a), who studied the macroeconomic dynamics of a model with an endogenous determination of corporate debt and firms' retention ratio. He observes the existence of multiple equilibria with unstable equilibrium at a higher level of the corporate debt-capital and retention ratios. However, this result requires that the exogenous interest rate be high in the model. In our model, however, the level of interest rate is irrelevant to the existence of multiple equilibria. However, a higher interest rate brings the stable and unstable equilibria closer as in Charles.\textsuperscript{15}

6.2 The Case of \( d_F = 0 \)

Now we consider the opposite case of consumer debt only, with no corporate debt case. Recall the consumption function of workers who can borrow to finance consumption expenditures:

\[
C_W = \theta(\overline{D_W} - D_W) +WL - iD_W
\]  \hspace{1cm} (41)

The rest of consumption is by rentiers, who now earn income only on consumer debt:

\[
C_R = (1 - s_R)iD_W
\]  \hspace{1cm} (42)

Since corporations are not engaged in debt finance, all investment must be finance by internal funds:

\[
I = \Pi_F
\]  \hspace{1cm} (43)

also, in this case, all profits are retained by firms so that \( \Pi_F = \Pi \). Our representation of investment behavior changes in a corresponding way:

\[
g_K = \kappa_0 + \kappa_r r_F
\]
\[
= \kappa_0 + \kappa_r r
\]
\[
= \kappa_0 + \kappa_r \tau u
\]  \hspace{1cm} (44)

Note that \( r = r_F \) since there is no corporate debt.

Temporary equilibrium is again characterized by \( Y = C + I \):

\[
Y = C_W + C_R + I
\]  \hspace{1cm} (45)

where

\[
C_W + C_R = \theta(\overline{D_W} - D_W) +WL - iD_W + (1 - s_R)iD_W
\]
\[
= \theta(\overline{D_W} - D_W) +WL - s_RiD_W
\]  \hspace{1cm} (46)

and

\[
I/K = \kappa_0 + \kappa_r \tau u
\]  \hspace{1cm} (47)

but his paper do not have much discussion on the possibility. His sufficient condition for a steady state at a positive leverage level to be stable is incorrect as well; the condition is only necessary, not sufficient.

\textsuperscript{15}The direct comparison with Charles (2008a) in terms of the dynamic results requires caution since his dynamics are two-dimensional, while ours in this section are one-dimensional.
Table 5: Short-Run Comparative Statics without Corporate Debt (u, r, and gK)

<table>
<thead>
<tr>
<th>u</th>
<th>r</th>
<th>gK</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>i</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>dW</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>π</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>sR</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

So normalizing by the capital stock and substituting for consumption and investment behavior we get

\[
u = \frac{1}{\pi(1 - \kappa_r)}[\kappa_0 + \theta(d_W - d_W) - is_Rd_W]
\]

(48)

The reduced form solutions for r and gK follow immediately:

\[
r = \frac{1}{(1 - \kappa_r)}[\kappa_0 + \theta(d_W - d_W) - is_Rd_W]
\]

(49)

\[
gK = \frac{1}{(1 - \kappa_r)}[\kappa_0 + \kappa_r\theta(d_W - d_W) - is_R\kappa_Rd_W]
\]

(50)

The comparative statistic results are mostly preserved (see table 5). A notable difference is that an increase the in interest rate has an unambiguously negative effect on u and r. An increase in the interest rate implies an income transfer from low-saving agents (workers) to high-saving agents (rentiers). This has a depressing effect on demand and hence on the accumulation rate in this demand-driven growth model.

Next we turn to consumer debt. From the definition of d_W, we see that

\[
\dot{d}_W = \frac{\dot{D}_W}{K} - \frac{\dot{K}D_W}{K^2}
= \frac{\theta(D_W - d_W)}{K} - \frac{I D_W}{K^2}K
\]

(51)

Substituting our reduced form for gK we get

\[
\dot{d}_W = \theta(d_W - d_W) - \frac{1}{(1 - \kappa_r)}[\kappa_0 + \kappa_r\theta(d_W - d_W) - is_R\kappa_Rd_W]d_W.
\]

\[
= \theta(1 - \kappa_r - \kappa_r dW)(d_W - d_W) - \frac{1}{1 - \kappa_r}[\kappa_0 - is_R\kappa_Rd_W]d_W.
\]

(52)

\[
= \frac{1}{(1 - \kappa_r)}[\theta(d_W - d_W)[1 - \kappa_r(1 + d_W)] - \kappa_0d_W + \kappa_ris_Rd_W^2]
\]
If we solve for the equilibrium points directly from here, we obtain

\[
d_W = \frac{1}{2\kappa_r(\theta + i\hat{s}_R)}\left\{\left[(1 - \kappa_r)\theta + \kappa_0 + \kappa_r\theta d_W\right]^2 - 4\kappa_r(\theta + i\hat{s}_R)(1 - \kappa_r)\theta d_W\right\}^{1/2}
\]

Assume that the value of \(d_W\) are not imaginary numbers. In other words, \([(1 - \kappa_r)\theta + \kappa_0 + \kappa_r\theta d_W]^2 > 4\kappa_r(\theta + i\hat{s}_R)(1 - \kappa_r)\theta d_W}.^\text{16} Then we observe that both steady state values of \(d_W\) are real numbers and positive.

\[
\frac{\partial}{\partial d_W} (\frac{dd_W}{dt}) = \frac{1}{(1 - \kappa_r)} \left[-(1 - \kappa_r)\theta - \kappa_0 - \kappa_r\theta d_W + 2\kappa_r(\theta + i\hat{s}_R)d_W\right]
\]

It is easy to see that a lower value of the consumer debt-capital ratio corresponds to a stable steady state and a higher value of the consumer debt-capital ratio corresponds to an unstable equilibrium. Figure 4 depicts the dynamics. It is interesting to note that the stable equilibrium exists at a positive level of consumer debt. This is in contrast with the dynamics with only corporate debt in the previous section. (Recall that the stable steady state exists only at the zero level of corporate debt, and the unstable steady state exists at a positive level of corporate debt in the previous section.) A certain positive level of consumer debt is sustainable in the model with only consumer debt. However, no positive level of corporate debt is sustainable in the model with only corporate debt.

\textsuperscript{16}With the following parameter values, we confirmed this condition: \(\theta = 0.3, s_r = 0.3, i = 0.1, \kappa_r = 0.6, \kappa_0 = 0.6, d_W = 0.5\).
6.3 Dynamics of Consumer and Corporate Debt

Even the simple models with only one kind of debt (consumer or corporate) in the previous sections produces multiple steady states. However, the results are contrasting. The model with only corporate debt produces a stable steady state at a zero level of corporate debt, but the model with only consumer debt produces a stable steady state with a positive level of consumer debt (both under reasonable restrictions). We now analyze the macroeconomic dynamics of the model including both corporate and consumer debt, and compare the results with the previous ones to understand the impact of including both corporate and consumer debt in the same model. Recall that we used the capital financing constraint in the transactions flows matrix \( \dot{D}_F = I - \Pi_F \) to conclude in (32) that

\[
\dot{d}_F = g_K - r_F - g_K d_F
\]  

(55)

In the case with no consumer debt, we were immediately able to produce (34), since in that case \( g_K - r_F = s_Ri d_F \). But now rentiers accumulate two kinds of claims. We use our temporary equilibrium solutions (24) and (25) to conclude that

\[
g_K - r_F = \frac{1}{1 - \kappa_r} \left[ \kappa_0 + \kappa_r \theta (d_W - d_W) - \kappa_r s_Ri(d_W + d_F) \right]
\]

\[
- \frac{1}{1 - \kappa_r} \left[ \theta(d_W - d_W) - s_Ri(d_W + d_F) + \kappa_0 \right]
\]

\[
= s_Ri(d_W + d_F) - \theta(d_W - d_W)
\]

(56)

Similar to the previous simpler case of no consumer debt, this has a natural interpretation: when capital accumulation exceeds available internal funds, it must be financed by borrowing the saving of rentiers, which also finances consumption borrowing. Substituting (56) into (55) we get

\[
\dot{d}_F = s_Ri(d_W + d_F) - \theta(d_W - d_W) - g_K d_F
\]

(57)

(Note that we have not yet completely reduced this, as it is still expressed in terms of \( g_K \), which is endogenous.)

Next we turn to consumer debt. From the definition of \( d_W \), we see that

\[
\dot{d}_W = \frac{\dot{D}_W}{K} - \frac{\dot{K} D_W}{K^2}
\]

\[
= \frac{\theta(D_W - D_W)}{K} - \frac{I D_W}{K}
\]

\[
= \theta(d_W - d_W) - g_K d_W
\]

(58)

Combining (57) and (58) we note

\[
\dot{d}_F + \dot{d}_W = [s_Ri(d_W + d_F) - \theta(d_W - d_W) - g_K d_F] + [\theta(d_W - d_W) - g_K d_W]
\]

\[
= (s_Ri - g_K)(d_F + d_W)
\]

(59)

Recalling our discussion of (34), this again suggests two different kinds of steady states:
one where \( s_R^i = g_K \), and one where \( d_F + d_W = 0 \). The first is a natural steady-state condition, which just says that steady-state debt must grow at the same rate as the steady-state capital stock. Note that the steady-state accumulation rate is solely determined by the saving behavior of rentiers and the exogenous interest rate. The second condition implies that the pure rentiers cease to exist at this steady state: the steady state values of the variables \( d_W \) and \( d_F \) have the same magnitude, but opposite signs.\(^{17}\)

Substituting the equilibrium expressions for the accumulation rate (25) into (57) and (58), we produce the following reduced-form equations of motion for \( d_W \) and \( d_F \):

\[
\begin{align*}
\dot{d}_F &= \frac{1}{(1 - \kappa_r)}[(\kappa_r - 1 - d_F \kappa_r)\theta(d_W) - d_W) + i\sigma_R(1 - \kappa_r + d_F \kappa_r)(d_W + d_F) - d_F \kappa_0] \\
\dot{d}_W &= \frac{1}{(1 - \kappa_r)}[\theta(d_W) - d_W)(1 - \kappa_r(1 + d_W)] - \kappa_0 d_W + \kappa_r i\sigma_R d_W(d_W + d_F)]
\end{align*}
\]

6.3.1 Phase Diagram

For a more detailed characterization of the dynamics, we will look at the nullclines of the equations of motion, and Jacobians around steady states. The nullcline \( \dot{d}_F = 0 \) is

\[
d_W = \frac{1}{(i\sigma_R + \theta)(1 - \kappa_r + d_F \kappa_r)} - (1 - \kappa_r)\theta d_W + d_F [\theta d_W \kappa_r - i\sigma_R(1 - \kappa_r + d_F \kappa_r) + \kappa_0]
\]

The nullcline \( \dot{d}_W = 0 \) is

\[
\dot{d}_F = -\frac{1}{\kappa_r i\sigma_R}[(1 - \kappa_r)\theta d_W - \kappa_r \theta d_W - \theta + \kappa_r(1 + d_W)\theta - \kappa_0 + d_W \kappa_r i\sigma_R]
\]

The Jacobian of the system is given by

\[
J(d_F, d_W) = \begin{bmatrix} H_{dF} & H_{dW} \\ G_{dF} & G_{dW} \end{bmatrix}
\]

where

\[
H_{dF} = \frac{1}{(1 - \kappa_r)}[-\kappa_0 - \kappa_r \theta(d_W) - d_W) + (1 - \kappa_r)i\sigma_R + \kappa_r i\sigma_R(d_W + 2d_F)]
\]

\[
H_{dW} = \frac{(\theta + i\sigma_R)(1 - \kappa_r + \kappa_r d_F)}{(1 - \kappa_r)}
\]

\[
G_{dF} = \frac{1}{(1 - \kappa_r)}d_W \kappa_r i\sigma_R
\]

\[
G_{dW} = \frac{1}{(1 - \kappa_r)}[(\kappa_r - 1)\theta - \kappa_0 - \kappa_r \theta(d_W - 2d_W) + \kappa_r i\sigma_R(2d_W + d_F)]
\]

\(^{17}\)Mathematically it could be the case that \( d_W \) is negative and same magnitude with \( d_F \). In other words, workers become rentiers, lending to firms. This is not the case as confirmed by simulations results with a reasonable range of parameters.
Let us look at the the the steady state which is characterized by the following condition, $d = d_W + d_F = 0 \rightarrow d_W = -d_F$. These points correspond to the stable equilibrium point A and unstable equilibrium point B in the phase diagram in Figure 5.\textsuperscript{18} Evaluating the self-feedback components of the Jacobians, equations (65) and (68), at the equilibrium point:

\begin{align*}
H_{d_F} &= \frac{1}{(1 - \kappa_r)}[-\kappa_0 - \kappa_r \theta (\overline{d_W} - d_W) + (1 - \kappa_r)is_R - \kappa_r is_R d_W] \\
G_{d_W} &= \frac{1}{(1 - \kappa_r)}[(\kappa_r - 1) \theta - \kappa_0 - \kappa_r \theta (\overline{d_W} - 2d_W) + \kappa_r is_R d_W]
\end{align*}

(69)  

(70)

We observe that the lower the level of $d_W$ is, the more likely both the self-feedback components and the trace are negative. And the higher the level of $d_W$ is, the more likely they are to be positive. More formally,

\[
\frac{\partial \text{Trace}}{\partial d_W} = 3\kappa_r \theta > 0
\]

(71)

The above information about the trace is consistent with the phase diagram in Figure 5. Steady state A, which is at a lower level of $d_W$, is a stable equilibrium, while the steady state C, which is at a higher level of $d_W$, is an unstable equilibrium.

Let us now examine the steady state characterized by the condition $g_K = is_R$. Using the\textsuperscript{18}With the following parameter values, we confirmed the phase diagram: $\theta = 1, s_r = 0.3, i = 0.1, \kappa_r = 0.6, \kappa_0 = 0.6, \overline{d_W} = 0.5$.
condition and equation (58), the steady state value of $d_W$ is:

$$d_W = \frac{\theta d_W}{i\bar{s}_R + \theta}$$  \hspace{1cm} (72)

Using equation (72), and the condition $g_K = i\bar{s}_R$, the steady state value of $d_F$ is calculated from equation (25) as:

$$d_F = \frac{1}{\kappa_r i\bar{s}_R} [\kappa_0 - (1 - \kappa_r) i\bar{s}_R]$$  \hspace{1cm} (73)

This point is represented by point B in phase diagram 5. At this point we evaluate the Jacobians,

$$H_{d_F} = \frac{1}{(1 - \kappa_r)} [\kappa_0 - (1 - \kappa_r) i\bar{s}_R] > 0$$  \hspace{1cm} (74)

$$G_{d_W} = \frac{1}{(1 - \kappa_r)} [(\kappa_r - 1) \theta + \kappa_r \theta \bar{d}_w - (1 - \kappa_r) i\bar{s}_R] \leq 0$$  \hspace{1cm} (75)

The trace and determinant of the system are given by

$$Trace = \frac{1}{(1 - \kappa_r)} [\kappa_0 - (1 - \kappa_r) i\bar{s}_R - (1 - \kappa_r)(\theta + i\bar{s}_R) + \kappa_r \theta \bar{d}_w]$$

$$Det = H_{d_F} G_{d_W} - H_{d_W} G_{d_F}$$  \hspace{1cm} (76)

This steady state is unstable. If $G_{d_W} > 0$, the trace is positive. The steady state is unstable. (It could be unstable in the sense of saddle dynamics if the determinant is negative as well.) If $G_{d_W} < 0$, the determinant of the Jacobian must be negative. (Recall that $H_{d_W}, G_{d_F} > 0$ at positive steady state points.) In other words, the steady state must be a saddle point without regard to the sign of trace. The phase diagram in Figure 5 illustrates this steady state is a saddle point.

The key element for the unstable dynamics in the above analysis lies in the the investment financing behavior of the firms. Combining the financial constraint in the transaction matrix (table 2) with the investment behavior,

$$dD_F/dt = I - (\Pi - iD_F) = \kappa_0 K + \kappa_r (\Pi - iD_F) - (\Pi - iD_F) = \kappa_0 K + (\kappa_r - 1) \Pi_F$$  \hspace{1cm} (77)

and normalizing by the capital stock

$$(dD_F/dt)/K = \kappa_0 + (\kappa_r - 1) r_F$$  \hspace{1cm} (78)

An increase in $d_F$ reduces the retained earning rate $r_F$, which induces the firm to borrow more. However, the increase in the capital stock is faster than the increase in corporate debt. We therefore observe a stable region at high levels of corporate debt. However, when the level of corporate debt is too high, debt service payments could eventually become higher than gross profits, and therefore $r_F(= r - iD_F)$ is negative. In this case, firms will have to
borrow just to make debt service payments, and debt grows faster. This sets off a vicious cycle. This process is closely related to Minsky’s ‘Ponzi’ state of firms, which refers to the case where the firms’ cash flow is not sufficient to cover the interest payments on the firms’ outstanding debt. The process is symmetric. A reduction in corporate debt would increase the retained earnings rate. This would increase the investment rate, which would induce higher retained earnings. Furthermore, a higher investment rate translates into more employment, and hence further demand injections via increases in workers’ consumption. Eventually the firms’ profit is high enough to pay back their debt, and \( \frac{dD_F}{dt} \) is negative in equation (78). This process continues until firms become lenders to workers, and pure rentiers cease to exist. Such an environment is represented by the point A in the phase diagram, Figure 5.

6.4 Comparative Dynamics

In this section, we perform several thought experiments to understand the impact of consumer debt on macroeconomic stability. The differential form of the equations of motion, equations (60) and (61), around the stable steady state point is

\[
J(d_F, d_W) \left[ \frac{dd_F}{dd_W} \right] + \left[ \frac{s_R(1 - \kappa_r + d_F \kappa_r)(d_W + d_F)}{d_W \kappa_r s_R(d_W + d_F)} \right] \frac{di}{dW} \\
+ \left[ \frac{i(1 - \kappa_r + d_F \kappa_r)(d_W + d_F)}{d_W \kappa_r i(d_W + d_F)} \right] \frac{ds_R}{dW} - \left[ \frac{d_F}{d_W} \right] \frac{d\kappa_0}{dW} \\
\frac{(\kappa_r - 1 - d_F \kappa_r)\theta}{\theta - \theta \kappa_r (1 + d_W)} \frac{dd_W}{dW} = 0
\]

At the stable steady state point A, the coefficient terms for \( di \) and \( ds_R \) become zero. In other words, the interest rate and saving behavior of rentiers do not have an effect on the stable steady-state values of \( d_W \) and \( d_F \). The comparative dynamics results with regard to \( d_W \) and \( \kappa_0 \) are more complicated to pin down.²⁰

To evaluate the comparative dynamics of \( u, r, r_F, \) and \( g \), we calculate the endogenous variables at the stable steady state.²¹ By substituting \(-d_W\) for \( d_F \), we obtain the following equations:

\[
u = \frac{1}{\pi(1 - \kappa_r)} \{ \theta d_W - [\theta + i(1 - \kappa_r)]d_W + \kappa_0 \} \quad \text{(80)}
\]

\[
r = \frac{1}{(1 - \kappa_r)} \{ \theta d_W - [\theta + i(1 - \kappa_r)]d_W + \kappa_0 \} \quad \text{(81)}
\]

²⁰In more realistic scenario, (which is not shown explicitly in our model dynamics), firms would have to use their existing assets (capital stock in our model) to make debt service payments. The declining capital stock must be accompanied by declining employment, and hence further reduction in aggregate demand. This will additionally reduce the firm’s profits, thereby accelerating the decline of the capital stock.

²¹The variable \( g \) denotes the economic growth rate. At the steady state the economic growth rate is equal to the accumulation rate.
\[ r_F = \frac{1}{(1 - \kappa_F)}[\kappa_0 + \theta(d_{W} - d_{W})] \]  
(82)

\[ g_K = \frac{1}{(1 - \kappa_r)}[\kappa_0 + \kappa_r\theta(d_{W} - d_{W})] \]  
(83)

From the above equations, the comparative dynamics results are calculated. See appendix A. for the computation.

Table 6: Long-Run Comparative Dynamics \((d_F, d_W, u, r, r_F, \text{ and } g)\)

<table>
<thead>
<tr>
<th>(d_F)</th>
<th>(d_W)</th>
<th>(u)</th>
<th>(r)</th>
<th>(r_F)</th>
<th>(g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>(d_{W})</td>
<td>-</td>
<td>+</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>(\kappa_0)</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>(s_R)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6 summarizes the comparative dynamics results. An increase in the consumer credit target has an ambiguous effect on the steady-state values of \(u, r, r_F, \text{ and } g\). Although the increase in borrowing by workers implies higher consumption, this is accompanied by a shift in income from workers to renters, who are higher saving agents.\(^{22}\) As equations (A.4) and (A.7) in appendix A. indicate, the effect will depend on how responsive the actual \(d_{W}\) is when there is an increase in \(d_{W}\). If the increase in the actual \(d_{W}\) is larger than the increase in \(d_{W}\), retained earning and economic growth rates will be reduced. It is notable that an increase in the consumer credit target shifts the saddle equilibrium point from B to D in figure 6. This implies that the stable region in the phase diagram expands. A higher consumer credit target (and hence higher borrowing) helps to sustain macroeconomic stability according to this result.\(^{23}\) For example, the macroeconomic trajectory at point E in the diagram becomes sustainable when there is an increase in the consumer credit limit. Consumer debt can sustain consumption, and hence demand, at a higher level relative to the case without consumer debt.

An increase in the interest rate has a negative effect on the stable steady-state values of the capacity utilization and profit rate, but not the retained earnings and growth rates. The reason is that, at this steady-state value, firms are also renters. The increase in the interest rate reduces the capacity utilization rate since this would reduce the worker’s overall consumption. However, it does not have an effect on retained earnings, and hence the growth rate, due to the interest income of firms. Since there are no pure renters at this steady state point, the saving propensity of renters does not affect any of the variables.

\(^{22}\)A similar result was obtained in Dutt (2006), who dealt only with consumer debt in a neo-Kaleckian growth model.

\(^{23}\)To understand the intuition, consider a macroeconomic trajectory at the point B. At that point, the growth rate of economy is just big enough to cover both consumer and corporate debt service payments; this growth rate also keeps the same rate of investment and consumption. A sudden increase in the target consumer credit limit will induce a higher capacity utilization rate. This higher capacity utilization rate generates higher retained earnings. The capital stock grows more quickly in response, driving \(d_F\) and \(d_W\) lower yet.
We note that both the interest rate and the saving propensity of rentiers can have a significant effect on macroeconomic stability. From the phase diagram in Figure 7, we observe that an increase in either the rentiers’ saving propensity or the interest rate shifts the saddle point from B to C. In other words, either a higher saving propensity of rentiers or a higher interest rate reduces the stable region in the positive space of \((d_F, d_W)\). For example, starting from point D the economy would have a stable trajectory converging to point A when B is the saddle point, but would have an unsustainable trajectory when C is the saddle point.\(^{24}\) This has an interesting policy implication. Monetary policy, via a change in the interest rate, could therefore have destabilizing effect on the macroeconomic trajectory. Raising interest rates in response to increased economic activity or inflation could have a severe consequence.

An increase in the ‘state of confidence’ has a stimulus effect on the long-run steady-state values of \(u, r, r_F,\) and \(g\). These are conventional results from a demand-driven growth model. From the phase diagram in Figure 8, we observe that an increase in state of confidence shifts the saddle point from point B to D. A higher state of confidence induces a larger stable region in the positive space of \((d_F, d_W)\).\(^{25}\) Conversely a decrease in the state of confidence

\(^{24}\)To understand the intuition, consider a macroeconomic trajectory at point B. At that point, the growth rate of economy is just big enough to cover both consumer and corporate debt service payments and to keep the same rate of investment and consumption. When there is a sudden decrease in interest rate, the growth rate is larger than the debt growth rate, which in turn generates higher internal funds and wages. Therefore, capital stock grows more quickly in response, driving \(d_F\) and \(d_W\) lower yet.

\(^{25}\)To understand the intuition, consider a macroeconomic trajectory at point B. At that point, the growth rate of economy is just big enough to cover both consumer and corporate debt service payments and to keep the same rate of investment and consumption. A sudden increase in the state of confidence will induce a higher accumulation rate, generating higher retained earnings. Capital stock grows more quickly in response,
can shrink the stable region. A stable macroeconomic trajectory at a higher level of both consumer and corporate debt (for example, at point E) can suddenly become unstable if there is sudden decrease in the state of confidence.

In summary, our comparative dynamics results suggest that an increase in the consumer credit target parameter enhances the stability of the system (in the sense of increasing the stable region). Consumer debt can sustain the macroeconomic trajectory, which may not be possible without consumer debt, because it provides an additional source of consumption finance for workers. It helps to sustain the demand side. However, this can make the system more vulnerable to changes in other structural parameters, such as the state of confidence, the interest rate, and the saving propensity of rentiers. An increase in the interest rate, rentiers’s saving propensity, or a reduction in the state of confidence can turn a sustainable macroeconomic trajectory into an unsustainable one. The impact may be more severe if the initial level of leverage is high, which could possibly lead to a deeper recession.

The key element for unstable dynamics lies in the firms’ borrowing behavior. There is no enforcement mechanism for firms to pay back their debt, even if the debt reaches an excessively high level. The same problem can be addressed from the lender’s point of view. High leverage ratios of firms should be a main determining factor of rentiers’ decisions on lending to such firms. The borrowers’ and lenders’ risks (in Minsky’s terms) are not appropriately incorporated in the model. This is a critical limitation of the previous efforts to incorporate Minsky’s financial instability into Kaleckian macro models (Lavoie, 1995; Hein, 2006; Charles, 2008a,b). Unfortunately, this important deficiency has not been recognized driving $d_F$ and $d_W$ lower yet.
to our knowledge.  

7 Incorporating Equity and Endogenous Retention Ratio

The analysis above yields some rather unusual results. Rentiers cease to exist at the stable steady state with a positive level of consumer debt. The reason for this result is that the economy has effectively only one asset in the rentiers' portfolio. (Recall that the interest rate for both corporate and consumer debt has been assumed to be same.) The firms' borrowing behavior is also unrealistic. There is no enforcement mechanism for firms to pay back their debt, even if the debt reaches an excessively high level. In this section, we address these problems by endogenizing the retention ratio. An endogenous retention ratio necessitates the introduction of equity, which then constitutes a second asset in rentiers's portfolio. The endogenous retention ratio also partially addresses our concern about the inadequate incorporation of borrowers' risks.

Table 7 shows the allocations across three types of agents of the four types of assets in this economy: capital ($K$), equity ($E$), loans to households ($D_W$), and loans to firms ($D_F$).

---

26This is also reason for the concurrence of 'paradox of debt' in post-Keynesian model with corporate debt (Lavoie, 1995; Hein, 2006). According to 'paradox of debt', higher interest rate will induce firms to cut down the investment to reduce the debt burden. However, this will reduce the profit of the firm and induce the further borrowing of the firms, generating unstable dynamics. 'Paradox of debt' occurs simply because there is no enforcement mechanism for firms to pay back their debt, when debt burden reaches an excessively high level.
Table 7: Balance sheet matrix

<table>
<thead>
<tr>
<th></th>
<th>Workers</th>
<th>Rentiers</th>
<th>Firms</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>0</td>
<td>0</td>
<td>$K$</td>
<td>$K$</td>
</tr>
<tr>
<td>Loans</td>
<td>$-D_W$</td>
<td>$D_W + D_F$</td>
<td>$-D_F$</td>
<td>0</td>
</tr>
<tr>
<td>Equities</td>
<td>$E$</td>
<td>$-E$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Net worth</td>
<td>$NW_W$</td>
<td>$NW_R$</td>
<td>$NW_F$</td>
<td>$K$</td>
</tr>
</tbody>
</table>

The column sums (across capital and loans) produce the net worth of each class of agent, while the row sums (across workers, rentiers, and firms) produce the net value of each class of asset.

Table 8 is the associated transaction matrix. In the case of firms, we distinguish between capital and current transactions. Following Charles (2008a), we abstract from new equity issue and the equity price for simplicity. Note that abstraction from the equity price rules out capital gains. A common interest rate $i$ applies to consumer and corporate debt. For the transaction matrix, we note that the sums across the rows must equal zero as a consistency condition. The columns also sum to zero reflecting budget constraints. Note that we do not assume that the measured net worth of firms is zero.

Table 8: Transaction Flow Matrix

<table>
<thead>
<tr>
<th>Firms</th>
<th>Workers</th>
<th>Rentiers</th>
<th>Current</th>
<th>Capital</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>$-C_W$</td>
<td>$-C_R$</td>
<td>$C_W + C_R$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>$W_rL$</td>
<td>$I$</td>
<td>$-W_rL$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Wages</td>
<td>$Dv$</td>
<td>$-(Dv + \Pi_F)$</td>
<td>$\Pi_F$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Firms’ profits</td>
<td>$iD_W$</td>
<td>$iD_W + iD_F$</td>
<td>$-iD_F$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Loan interest</td>
<td>$\tilde{D}_W$</td>
<td>$-(\tilde{D}_W + \tilde{D}_F)$</td>
<td>$\tilde{D}_F$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Change in loans</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Unlike in the previous sections, where the retention ratio was assumed to be its maximum value of 1, in this section, the retention ratio is endogenous and assumed to take a logistic functional form as,

$$s_f = \frac{\varepsilon}{1 + e^{-d_F}} \quad (84)$$

where $\varepsilon \leq 1$ and $e$ is the base of the natural logarithm (See figure 9). Note that the advantage of this specification is that the retention ratio has a reasonable upper bound $\varepsilon$. A lower value of $\varepsilon$ represents less adverse behavior of the firms regarding their indebtedness.\footnote{Initially the retention behavior employed by Charles (2008a) was specified as:}

$$s_f = \varepsilon_0 + \frac{\varepsilon_1}{d_F},$$

where both $\varepsilon_0$ and $\varepsilon_1$ are constant terms. The model dynamics with this retention behavior is not well
The specified logistic form of the endogenous retention ratio is not only mathematically convenient, but economically plausible. The implied behavior assumes that firms perceive a higher leverage position as a higher risk position. When there is an increase in the leverage ratio, firms' managers increase the retention ratio to preserve their financial position.

In this new version of the model, firms pay dividends, so gross profit is split between debt service, dividends, and retained earnings:

$$\Pi = Div \cdot iD_F + s_f \Pi_F$$  \hspace{1cm} (85)

where

$$\Pi_F = \Pi - iD_F$$  \hspace{1cm} (86)

Recall that $\Pi_F$ denoted retained earning in the previous chapter because there was no equity in the model. We now define $\Pi_F$ as net profit (gross profit minus debt service). Dividend payout behavior is modeled as

$$Div = (1 - s_f)\Pi_F = [1 - (\frac{\varepsilon}{1 + e^{-d_F}})]\Pi_F$$  \hspace{1cm} (87)

This behavior has been used in a number of recent works (Lavoie and Godley, 2002; Charles, 2008a; Skott and Ryoo, 2008).\(^{28}\) We now decompose the gross profit rate into the retained earning rate, dividend rate, and leverage ratio:

$$r = s_f r_F + (1 - s_f) r_f + iD_F$$  \hspace{1cm} (88)

The investment function with the endogenous retention ratio is

$$g_K = k_0 + k_r(\frac{\varepsilon}{1 + e^{-d_F}})(r_F)$$  \hspace{1cm} (89)

\(^{28}\)However, in our knowledge, the endogenous retention ratio is modeled as a logistic function for the first time in this paper.
The consumption function of the rentiers now includes the dividend payout.

\[
C_R = (1 - s_R)(iD_W + iD_F + Div)
\]

(90)

It would be equivalent to specify the saving behavior as \( S_R = s_R(iD_W + iD_F + Div) \). Once we have this behavior in place, using the rentiers’ column in the transaction flow matrix (8), we can characterize the saving behavior of the rentiers in the following way:

\[
\dot{D}_F + \dot{D}_W = s_R(iD_W + iD_F + Div)
\]

(91)

\[
\frac{\dot{D}_F + \dot{D}_W}{i(D_F + D_W) + Div} = s_R
\]

(92)

Workers do not hold any equity as seen in table 7. Therefore there is no modification in their behavior.

The temporary equilibrium values of endogenous variables are:

\[
u = \frac{1}{\pi [s_R + (1 - s_R - \kappa_r) \frac{\varepsilon}{1 + e^{-d_F}}]} \left[ \kappa_0 + \bar{d}_W - (\theta + s_R \bar{i})d_W ight. \\
+ \left. (1 - s_R - \kappa_r) \frac{\varepsilon}{1 + e^{-d_F}} \bar{i}d_F \right]
\]

(93)

\[
\tau = \frac{1}{[s_R + (1 - s_R - \kappa_r) \frac{\varepsilon}{1 + e^{-d_F}}]} \left[ \kappa_0 + \bar{d}_W - (\theta + s_R \bar{i})d_W ight. \\
+ \left. (1 - s_R - \kappa_r) \frac{\varepsilon}{1 + e^{-d_F}} \bar{i}d_F \right]
\]

(94)

\[
r_F = \frac{1}{[s_R + (1 - s_R - \kappa_r) \frac{\varepsilon}{1 + e^{-d_F}}]} \left[ \kappa_0 + \bar{d}_W - (\theta + s_R \bar{i})d_W - s_R \bar{id}_F \right]
\]

(95)

\[
s_{rF} = \frac{\varepsilon}{[(1 + e^{-d_F})s_R + (1 - s_R - \kappa_r)\varepsilon]} \left[ \kappa_0 + \bar{d}_W \\
\right. \\
\left. - (\theta + s_R \bar{i})d_W - s_R \bar{id}_F \right]
\]

(96)

\[
g_K = \kappa_0 + \kappa_r \left( \frac{1}{1 + e^{-d_F}} \right) \left[ s_R + (1 - s_R - \kappa_r) \frac{\varepsilon}{1 + e^{-d_F}} \right] \left[ \kappa_0 + \bar{d}_W \\
\right. \\
\left. - (\theta + s_R \bar{i})d_W - s_R \bar{id}_F \right]
\]

(97)

### 7.1 Short Run Comparative Statics

Comparative static results are presented in the table 9. Many of the results in the table 3 are preserved and intuition is also carried over. One of the main differences is that an increase in \( d_F \) has ambiguous effects on \( s_{rF} \) and \( g_K \). Higher corporate debt results in higher interest
Table 9: Short-Run Comparative Statics ($u$, $r$, $s_f r_F$ and $g_K$)

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$r$</th>
<th>$s_f r_F$</th>
<th>$g_K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_0$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$i$</td>
<td>?</td>
<td>?</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$d_F$</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$d_W$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{d}_W$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$s_R$</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$\pi$</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

payments for firms, cutting net profits. On the other hand it increases rentiers’ consumption via higher interest income. Furthermore firms will increase their retention ratio $s_f$. The net effect on retained earning and accumulation rates are therefore ambiguous. An increase in the retention ratio due to an increase in the exogenous parameter $\varepsilon$ has an ambiguous impact on the endogenous variables. Although an increase in $\varepsilon$ increases the retention ratio, the retained earning rate, and hence has a positive effect on accumulation rate, it also reduces the dividend income of the rentiers, thereby reducing rentiers’ consumption. The overall impact is therefore ambiguous.

7.2 Dynamics

The equation of motion for corporate debt-capital ratio is derived as follows:

$$
\dot{d}_F = \frac{\dot{D}_F}{K} - \frac{tK \dot{D}_F}{K} = \frac{I - s_f \Pi_F}{K} - \frac{tID_F}{K} = \frac{I - (\frac{\varepsilon}{1+e^{-d_F}})\Pi_F}{K} - \frac{tID_F}{K} = g_K - \left(\frac{\varepsilon}{1+e^{-d_F}}\right)t_F - g_K d_F
$$

Substituting the appropriate equations for the endogenous variables,

$$
\dot{d}_F = g_K - \left(\frac{\varepsilon}{1+e^{-d_F}}\right)r_F - g_K d_F = \kappa_0(1 - d_F) + \frac{(\kappa_r - 1 - \kappa_r d_F)}{s_R + (1 - s_R - \kappa_r)\frac{\varepsilon}{1+e^{-d_F}}} \left[\kappa_0 + \theta d_W - (\theta + s_R i)d_W - s_R d_F\right]
$$
The equation of motion for the consumer debt capital ratio is similarly derived:

\[
\begin{align*}
\dot{d}_W &= \theta(d_{\bar{W}} - d_W) - g_K d_W \\
&= \theta(d_{\bar{W}} - d_W) - d_W k_0 \\
&\quad - \kappa_r \left( \frac{\varepsilon}{1 + e^{-d_F}} \right) \left[ s_R + (1 - s_R - \kappa_r) \frac{d_W}{1 + e^{-d_F}} \right] \left[ k_0 + \theta d_{\bar{W}} - (\theta + s_R) d_W - s_R d_F \right]
\end{align*}
\]  

(100)

7.2.1 Phase Diagram

Due to the complexity of the equations of motion, we will rely on simulation methods for the analysis. Figure 10 presents the phase diagram of the model. It is notable that the model now possesses a stable steady state in the positive space of \((d_F, d_W)\). The key element for the unstable dynamics around the saddle point B in the phase diagram lies in the investment financing behavior of the firms. Combining the financial constraint in the transaction matrix (table 8) with the investment behavior,

\[
\begin{align*}
\frac{dD_F}{dt} &= I - s_f(\Pi - iD_F) = k_0 K + \kappa_r (\Pi - iD_F) - s_f(\Pi - iD_F) \\
&= k_0 K + (\kappa_r - 1)s_f\Pi_F
\end{align*}
\]  

(101)

and normalizing by capital stock

\[
\frac{(dD_F/dt)}{K} = k_0 + (\kappa_r - 1)s_f r_F
\]  

(102)

---

This phase diagram is obtained assuming the following parameter values: \(k_0 = 0.4, \theta = 1, \kappa_r = 0.4, \psi = 0.03, d_{\bar{W}} = 0.3, s_R = 0.3, \varepsilon = 0.7\). These parameter values are also our benchmark values for the later study on macroeconomic stability. With this parameter values, the system exhibits a nodal sink point at \((0.20035, 0.1889)\) and a saddle point at \((123.7474, 0.29762)\). We exclude some part of phase diagram to focus on the positive space of \((d_F, d_W)\).
When the level of corporate debt is too high, debt service payments could eventually become higher than net profits, and therefore $r_F = r - id_F$ as well as $s收益率$ are negative. When the net profit rate is negative, firms will have to borrow just to make debt service payments, and debt grows faster. This sets off a vicious cycle. Debt grows at an exponential rate. Note that rentiers continue to lend more to firms although profits are falling and the debt stock is continuously increasing. Although firms can increase the retention ratio to its maximum value as the debt stock grows, there is no mechanism that compels firms to stop borrowing and pay back principal even when existing debt become excessive relative to net profit.

We also observe that the stable saddle path which divides the stable and unstable regions is downward sloping. The reason is found in the negative relationship between the consumer debt-capital ratio and capacity utilization. A higher consumer debt-capital ratio means lower capacity utilization and a lower gross profit rate, all else being equal. The unstable dynamics become effective at a lower level of the corporate debt-capital ratio when the consumer debt-capital ratio is higher. This is captured by the downward sloping stable saddle path.

The unstable process is closely related to Minsky’s ‘Ponzi’ state of firms, which refers to the case where the firms’ cash flow is not sufficient to cover the interest payments on the firms’ outstanding debt. They have to borrow just to make debt service payments. The endogenous retention ratio cannot provide an enough of a damping mechanism since it is bounded by 1 even when the level of debt is too high.

### 7.3 Macroeconomic Stability

The impact of the underlying parameters on macroeconomic stability is depicted in figures 11, 12, and 13. The results are qualitatively similar to the parallel results without the endogenous retention ratio. The intuition is also similar in each case. Figure 14 depicts the comparative dynamics when firms become more frugal in their dividend policy and increase in their retained earnings (modeled here as an increase in $\varepsilon$). An increase in $\varepsilon$ means an increase in firms’ retention ratio, for any given level of $d_F$. This will reduce dividend income and hence the supply of funds available to consumer and firms (recall equation 91). Therefore, the level of debt at both steady states is lower than before, resulting in a smaller stable region.

#### 7.3.1 Emergence of Macroeconomic Crisis

This section is the core part of the analysis and deals with the emergence of macroeconomic crisis. Figure 15 presents the emergence of macroeconomic crisis when there is a large downward swing in the state of confidence to a negative level. Compare this phase diagram with the baseline phase diagram in figure 10. The stable steady state ceases to exist. The

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30 In more realistic scenario, (which is not shown explicitly in our model dynamics), firms would have to use their existing assets (capital stock in our model) to make debt service payments. The declining capital stock must be accompanied by declining employment, and hence further reduction in aggregate demand. This will additionally reduce the firm’s profits, thereby accelerating the decline of the capital stock.

31 The baseline values of parameters are: $\kappa_0 = 0.4, \theta = 1, \kappa_r = 0.4, \delta = 0.03, \delta_W = 0.3, s_R = 0.3, \varepsilon_0 = 0.7$.

32 One could obtain such phase diagram with the following parameter values: $\kappa_0 = -0.1, \theta = 1, \kappa_r = 0.4, \delta = 0.03, \delta_W = 0.3, s_R = 0.3, \varepsilon_0 = 0.7$. Nodal source (point B) is at (-1.2333, 4.7714) and saddle (point A) is at (0.091016, 0.35593).
Figure 11: The Effect of a Higher Credit Target

Figure 12: The Effect of a Higher State of Confidence
Figure 13: The Effect of a Higher Interest or Rentier’s Saving Rate

Figure 14: The Effect of a Higher Retention Ratio (An Increase in $\varepsilon$)
steady state \( A \) is a saddle point, and \( B \) is a nodal source, which both of which are unstable.\(^{33}\) This result is interestingly unique to the model with the logistic endogenous retention ratio.\(^{34}\) When there is a sudden change in the state of confidence to a negative level \((\kappa_0 < 0)\), the stable macroeconomic trajectory becomes an unstable one and explodes. (The direction of course will depend on the initial condition.) For example, consider the macroeconomic trajectory at point \( C \) in figure 15. Suppose it was initially a stable one. After the change in the state of confidence to a negative level, the macroeconomic trajectory at point \( C \) suddenly become unstable and the system explodes. To understand the intuition recall equation (102):

\[
\frac{(dD_F/\ dt)}{K} = \kappa_0 + (\kappa_\tau - 1) \sigma \tau_F
\]

The negative animal spirits term \((\kappa_0 < 0)\) means a pessimistic view of firms about economic prospects. They are now more willing to pay back the principal of debt \((dD_F/\ dt \) becomes negative), and use their existing assets (capital) for paying back the debt. There is a disinvestment process in the economy. The declining capital stock must be accompanied by declining employment, and hence further reductions in aggregate demand. This will additionally reduce the firm’s profits, thereby accelerating the decline of the capital stock. Due to the declining capital stock, we see that the consumer debt capital ratio also explodes.

Figure 16 depicts the emergence of an economic crisis due to a combination of shifts

\(^{33}\)With larger swing in the state of confidence to the negative level (for example, \(\kappa_0 = -0.5\)), the steady states completely cease to exists. In that case, the dynamics are essentially same with the one in the figure 15, except no existence of stable manifold (stable saddle path).

\(^{34}\)Mathematically, the model is structurally unstable. If the general characteristics of dynamics is preserved when there is a small perturbation of a dynamical system, the system is a structurally stable one. A well-known structurally unstable model is Richard M. Goodwin’s growth cycle model (Goodwin, 1967). For the precise mathematical definition, see the classic Hirsch and Smale (1974).
in behavioral parameters: higher saving of rentiers and a higher interest rate.\footnote{For example, one could obtain such a phase diagram with the following parameter values: $\kappa_0 = 0.4, \theta = 1, \kappa_r = 0.4, \iota = 0.7, d_W = 0.3, s_R = 0.7, \varepsilon_0 = 0.7$.} In this case, there is no steady state in a reasonable range of positive debt values.\footnote{There is an unstable steady state, spiral source at (-2.5094, 32.09), using the parameters in the previous note. This is not shown in the phase diagram.} This result is also unique to the model with the logistic endogenous retention ratio. According to this result, when this sudden change in a combination of behavioral parameters occurs, the stable macroeconomic trajectory becomes an unstable one and explodes.\footnote{Obviously there can be many different combinations of the changes in behavioral parameters, generating similar results. For example, one could obtain such phase diagram with the following parameter values: $\kappa_0 = 0.1, \theta = 1, \kappa_r = 0.4, \iota = 0.5, d_W = 0.3, s_R = 0.5, \varepsilon_0 = 0.7$. In this case, there is an unstable steady state, spiral source at (-1.7568, 11.3047).} For example, consider the macroeconomic trajectory at point A in Figure 16. Suppose it was initially a stable one. After the changes in parameters, we observe that it suddenly becomes an unstable macroeconomic trajectory and explodes. To understand the intuition, again recall equation (102):  

$$(dD_F/dt)/K = \kappa_0 + (\kappa_r - 1) s_F r_F$$

An increase in rentier's saving and the interest rate increases demand leakages. In this demand driven growth model, this will result in a reduction of net profits ($r_F$). This will induce firms to borrow more (assuming that $\kappa_r - 1 < 0$). This is not sustainable. Debt service payments eventually become higher than net profits, and hence $r_F(= r - id_F)$ becomes negative. Firms will now have to borrow just to make debt service payments, and debt grows faster. This in turn generates further borrowing for debt service payments, eventually generating exploding dynamics.
8 Concluding Remarks

Psychological factors played a central role in Keynes' explanation of the business cycle (trade cycle in his words) and economic crisis (see Keynes, 1936, chap. 22). Such psychological factors are parameterized as $\kappa_0$ in an admittedly ad hoc fashion in this model. A reduction in the state of confidence to a negative level creates complete macroeconomic instability. A stable macroeconomic trajectory becomes suddenly an unstable one, and explodes. Economic crisis and recovery induced by the downswings and upswings of state of confidence could be conceptualized as debt dynamics shifting between the ones in figures 10 and 15. This also has a critical policy implication. The state of confidence is not easily measurable. It is also a difficult variable to control via policy intervention. This points out an inherent policy difficulty for preventing and/or recovering an economy from a debt-driven economic downturn triggered by faltering confidence.

\[ \ldots \text{it is not so easy to revive the marginal efficiency of capital, determined, as it is, by the uncontrollable and disobedient psychology of the business world. It is the return of confidence, to speak in ordinary language, which is so insusceptible to control in an economy of individualistic capitalism. (Keynes, 1936, p. 317)} \]

Other types of behavioral shifts can also create an economic crisis. For example, higher interest rate and more prudent behavior of rentiers can transform the macroeconomic system into a completely unstable one as shown in figure 16. As Minsky forcefully argued, a capitalist macroeconomic system with debt possesses inherent or latent instability.

We have investigated the model with the fixed retention rate at its maximum value (section 6.3) as well as with an endogenous retention ratio modeled as a logistic functional form (7). In both cases, consumer debt can sustain the macroeconomic trajectory, which may not be possible without consumer debt, because it provides an additional source of consumption for workers. It helps to sustain the demand side. An increase in the consumer credit target enhances the stability of the system in the sense of increasing the stable region (recall figures 6 and 11). However, this can make the system, in a sense, more vulnerable to a change in other structural parameters, such as the state of confidence, the interest rate, and the saving propensity of rentiers. An increase in the interest rate or rentiers’ saving propensity, or a reduction in the state of confidence, can turn a sustainable macroeconomic trajectory into an unsustainable one. The impact may be more severe if the initial level of leverage is high (created by debt-financed workers’ consumption), which could possibly lead to a deeper recession.

The key element for the unstable dynamics lies in the firms’ borrowing behavior. There is no enforcement mechanism to make firms pay back their debt principal. The endogenous retention ratio does not address this limitation properly. Although firms can increase the retention ratio to its maximum value as the debt stock grows, it does not imply a mechanism which allows firms to pay back principal when existing debt becomes excessive relative to net

\[ \text{38Minsky's work of course build upon this idea, unlike the modern orthodox economics. For the other works in this direction, see Galbraith (1955), Kindleberger and Aliber (2005), and Akerlof and Shiller (2009).} \]
profits. The same problem can be addressed from lender’s point of view. High leverage ratios of firms should be a main determining factor of rentiers’ decisions on lending to such firms. The borrowers’ and lenders’ risks (in Minsky’s terms) are not appropriately incorporated in the model. This is a critical limitation of the previous efforts to incorporate Minsky’s financial instability hypothesis into neo-Kaleckian macro models (Lavoie, 1995; Hein, 2006; Charles, 2008a,b). To our knowledge, this important limitation has not even been acknowledged in the literature. We plan to address this limitation in our future research.

References


A. Comparative Dynamics

In this section the comparative dynamic results for table 6 are formalized using the derivatives from equations (80), (81), (82), and (83). Only the results for $u$ and $r_F$ are reported here.\(^{39}\)

\[
\begin{align*}
\frac{\partial u}{\partial i} &= \frac{1}{\pi(1 - \kappa_r)}[-(1 - \kappa_r)d_W] < 0 \\
\frac{\partial u}{\partial \kappa_0} &= \frac{1}{\pi(1 - \kappa_r)}[-[\theta + i(1 - \kappa_r)]\frac{\partial d_W}{\partial \kappa_0} + 1] > 0 \\
\frac{\partial u}{\partial d_W} &= \frac{1}{\pi(1 - \kappa_r)}[\theta - [\theta + i(1 - \kappa_r)]\frac{\partial d_W}{\partial d_W}] \leq 0 \\
\frac{\partial u}{\partial s_R} &= \frac{1}{\pi(1 - \kappa_r)}\{-[\theta + i(1 - \kappa_r)]\frac{\partial d_W}{\partial s_R}\} = 0 \\
\frac{\partial r_F}{\partial i} &= \frac{1}{(1 - \kappa_r)}(-\theta \frac{\partial d_W}{\partial i}) = 0 \\
\frac{\partial r_F}{\partial d_W} &= \frac{1}{(1 - \kappa_r)}(\theta - \theta \frac{\partial d_W}{\partial d_W}) \leq 0 \\
\frac{\partial r_F}{\partial \kappa_0} &= \frac{1}{(1 - \kappa_r)}(1 - \theta \frac{\partial d_W}{\partial \kappa_0}) > 0 \\
\frac{\partial r_F}{\partial s_R} &= \frac{1}{(1 - \kappa_r)}(-\theta \frac{\partial d_W}{\partial s_R}) = 0.
\end{align*}
\]

To evaluate the shift of the graphs in section 6.4, we compute the following derivatives from the equations of motion. We observe that the signs of the derivatives are largely

\(^{39}\)The results for $r$ must be same as those for $u$, and the results for $g$ must be same as those for $r_F$. 
determined by the signs and relative magnitudes of $d_W$ and $d_F$.

\[
\frac{\partial d_W}{\partial i} \bigg|_{d_F=0} = -\frac{s_R(d_W + d_F)}{\theta + i s_R} \ll 0 \quad (A.10)
\]

\[
\frac{\partial d_F}{\partial i} \bigg|_{d_W=0} = -\frac{(d_W + d_F)}{i} \ll 0 \quad (A.11)
\]

\[
\frac{\partial d_W}{\partial s_R} \bigg|_{d_F=0} = -\frac{i (d_W + d_F)}{\theta + i s_R} \ll 0 \quad (A.12)
\]

\[
\frac{\partial d_F}{\partial s_R} \bigg|_{d_W=0} = -\frac{(d_W + d_F)}{s_R} \ll 0 \quad (A.13)
\]

\[
\frac{\partial d_W}{\partial d_W} \bigg|_{d_F=0} = \frac{\theta}{\theta + i s_R} > 0 \quad (A.14)
\]

\[
\frac{\partial d_F}{\partial d_W} \bigg|_{d_W=0} = -\frac{\theta [(1 - \kappa_r(1 + d_W)]}{d_W \kappa_r i} \ll 0 \quad (A.15)
\]

\[
\frac{\partial d_W}{\partial \kappa_0} \bigg|_{d_F=0} = \frac{d_W}{(1 - \kappa_r + \kappa_r d_F)(\theta + i s_R)} \ll 0 \quad (A.16)
\]

\[
\frac{\partial d_F}{\partial \kappa_0} \bigg|_{d_W=0} = \frac{1}{i \kappa_r s_R} > 0. \quad (A.17)
\]
### B. Notation

Table B.1: Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_W$</td>
<td>consumption by workers</td>
</tr>
<tr>
<td>$C_R$</td>
<td>consumption by rentiers</td>
</tr>
<tr>
<td>$C$</td>
<td>$C_W + C_R$ (total consumption: workers and rentiers)</td>
</tr>
<tr>
<td>$I$</td>
<td>gross private domestic investment.</td>
</tr>
<tr>
<td>$K$</td>
<td>capital stock</td>
</tr>
<tr>
<td>$W_r$</td>
<td>real wage</td>
</tr>
<tr>
<td>$W_n$</td>
<td>nominal wage</td>
</tr>
<tr>
<td>$L$</td>
<td>labor input into production (labor hired)</td>
</tr>
<tr>
<td>$i$</td>
<td>interest rate</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>firms' gross profits</td>
</tr>
<tr>
<td>$\pi$</td>
<td>the gross profit share ($\Pi/Y$)</td>
</tr>
<tr>
<td>$\Pi_F$</td>
<td>firms' retained earnings</td>
</tr>
<tr>
<td>$r$</td>
<td>gross profit rate ($\Pi/K$)</td>
</tr>
<tr>
<td>$r_F$</td>
<td>retained earnings rate ($\Pi_F/K$)</td>
</tr>
<tr>
<td>$D_F$</td>
<td>the debt of firms (corporate debt)</td>
</tr>
<tr>
<td>$D_W$</td>
<td>the debt of wage earners (consumer debt)</td>
</tr>
<tr>
<td>$D_{W'}$</td>
<td>credit target of workers</td>
</tr>
<tr>
<td>$S_W$</td>
<td>the saving of wage earners</td>
</tr>
<tr>
<td>$S_R$</td>
<td>the saving of rentiers</td>
</tr>
<tr>
<td>$s_R$</td>
<td>rentiers' saving coefficient</td>
</tr>
<tr>
<td>$\Pi_F$</td>
<td>retained earnings (as accounting residual)</td>
</tr>
<tr>
<td>$NW$</td>
<td>net worth</td>
</tr>
<tr>
<td>$d_F$</td>
<td>$D_F/K$ ratio</td>
</tr>
<tr>
<td>$d_W$</td>
<td>$D_W/K$ ratio</td>
</tr>
<tr>
<td>$\bar{d}_W$</td>
<td>target $D_W/K$ ratio</td>
</tr>
<tr>
<td>$u$</td>
<td>the rate of capacity utilization</td>
</tr>
<tr>
<td>$\tau$</td>
<td>constant markup rate</td>
</tr>
<tr>
<td>$\kappa_0$</td>
<td>the state of business confidence</td>
</tr>
<tr>
<td>$\kappa_r$</td>
<td>the sensitivity of desired investment to the retained earning rate</td>
</tr>
<tr>
<td>$u$</td>
<td>utilization rate ($Y/K$)</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>maximum retention ratio</td>
</tr>
<tr>
<td>$g_K$</td>
<td>the desired investment rate</td>
</tr>
</tbody>
</table>